Estimating Driver Mandatory Lane Change Behavior on a Multi-lane Freeway

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ABSTRACT
The driver gap acceptance and rejection behavior during mandatory lane changes on a multilane freeway are analyzed in this paper. Gaps are accepted or rejected based on comparison with a minimum value generally defined as the critical gaps. Critical gaps are estimated based on the accepted and rejected gaps observed in the field. Driver behavior can be classified as consistent or inconsistent on the basis of gap rejection. For consistent driver behavior, it is assumed that the rejected gaps are shorter than the accepted gaps. This paper focuses on the estimation of critical gaps values and its distribution for consistent driver behavior. Critical gap, for consistent driver behavior is defined as the minimum value of gap above which the lane changer does not reject a gap to execute a lane change. Several gaps may be rejected prior to a gap being accepted, therefore, different types of rejected gaps can be utilized to estimate critical gaps. To systematically evaluate rejected gaps and propose the most suitable rejected gaps for use in estimating the critical gaps, rejected gaps were analyzed using the mean rejected, median rejected, and the largest rejected gaps less than the accepted gaps (LRLA). To model the consistent gap acceptance behavior of drivers i.e. the rejected gap is less than the accepted gap, LRLA is used in estimating the critical gaps. The values of critical time gaps were estimated using the maximum likelihood estimation method. This paper utilized the data collected by the NGSIM project on I-80 during both uncongested and congested traffic conditions.

Keywords: gap acceptance, gap rejection, critical gaps, lane change behavior, microscopic simulation models.

INTRODUCTION
Gap acceptance is an important component of lane change algorithms used in microscopic traffic simulation models. This paper presents an analysis of driver gap acceptance and gap rejection behavior using detailed vehicle trajectory data (1) and proposes statistical distributions for accepted and critical time gaps which can be utilized in modeling realistic mandatory lane change behavior of drivers in microscopic simulation models. Common terminologies used in this paper are defined in Table 1.

Researchers, in the past, have proposed methods to estimate critical time gaps for unsignalized intersections that are useful in estimating capacity and delay (2, 3). The Highway Capacity Manual (HCM) (4) defines a critical gap for an unsignalized intersection as the minimum time, in seconds, between successive major stream vehicles in which a minor-street vehicle can make a maneuver and enter the major street. To represent other gap acceptance situations such as lane changes on a freeway, merging or diverging on a freeway, or overtaking on a bi-directional road, the more generalized definition of critical gaps is the
minimum time between successive vehicles in which the subject vehicle can perform the intended maneuver (5). A minimum value of accepted gaps from a population, however, can not be considered a critical gap as it would only represent the gap acceptance behavior of an aggressive driver and would not represent the minimum acceptable gap of the driver population.

Critical gaps are modeled as random variables and different distributions have been proposed in the literature; Herman and Weiss (6) assumed an exponential distribution, Drew et. al. (7) assumed a log-normal distribution, and Miller (3) assumed a normal distribution. Similarly, lane change models in simulation software such as MITSIM (8), CORSIM (9), VISSIM (10), and SITRAS (11) use critical gaps in different ways. MITSIM assumes the distribution of critical gaps to have a lognormal distribution and takes into account the leading and trailing critical distance gaps. CORSIM defines critical gaps by using risk factors. The risk factor is defined by the rate of deceleration a driver applies to avoid collision if the leader brakes to a complete stop. Risk factors are evaluated for every lane change between the subject vehicle and the assumed leader and between the subject vehicle and the assumed follower. A risk factor is compared to an acceptable risk factor that depends on types of lane changes, drivers, and urgency of lane change. In VISSIM, the psychophysical model presents critical gaps as thresholds that depend on the relative speeds of the subject vehicle and the assumed leader and the subject vehicle and the assumed follower. In SITRAS, a critical gap is defined as the minimum acceptable gap and is equal to the sum of minimum safe constant gap and the product of a constant with relative speed.

Lane change maneuvers may reduce the freeway capacity under different traffic flow conditions. As critical gap distributions are used in lane change models as described above, using the realistic distribution of critical gaps is important in producing the frequency of lane changes close to real world conditions, close replication of driver behavior during lane changes, and freeway capacity estimations. The distribution of critical gaps, therefore, should be determined so it can be used in simulation models to have results that are in close agreement with the field data. Unrealistic results can lead to erroneous results about the characteristics of the facility being simulated.

Furthermore, in past studies for unsignalized intersections (2, 3), driver gap acceptance behavior have been categorized as consistent and inconsistent. Critical gaps, for unsignalized intersections, were estimated assuming consistent driver gap acceptance behavior. Consistent behavior assumes that drivers do not reject gaps that are larger than the accepted gaps. On freeways during an intended lane change, however, drivers reject a gap due to various reasons such as to avoid collision with the assumed leader and the assumed follower in the target lane, have not reached the intended location of lane change, late reaction to traffic signs, etc. The rejected gaps, therefore, may or may not be less than the accepted gaps, contrary to the assumption for consistent drivers at unsignalized intersections. The lane changes on a freeway are also in contrast to gap acceptance at unsignalized intersections where a driver comes to a complete stop before entering the major street. On freeways, a driver seeks a suitable gap while in motion and it is not possible to determine the exact point in space or time when a driver decides to make a lane change. As critical time gaps are a function of accepted and rejected time gaps, and there can be several rejected gaps before a gap is accepted, it is important to consider an appropriate value of a rejected gap to get an estimate of critical gaps that represent realistic driver behavior. To estimate the value of critical gaps, different rejected gaps are evaluated in this paper. To analyze the detailed
gap rejection behavior, the mean rejected, median rejected and the largest rejected gap less than the accepted gap (LRLA) are used. A previous paper (12) analyzed the largest rejected gap prior to a lane change which accounted for most reasons, if not all, for gap rejection.

The paper is organized as follows: first, the methodology is discussed, second, the field data and data reduction is presented, third the results from the application of the methodology to the field data sets are described and last, conclusions and recommendations are summarized. All gaps referred to, in this paper, are time gaps, unless otherwise stated.

METHODOLOGY
This paper analyzes NGSIM datasets, described in the next section, to determine the driver leading and trailing accepted and rejected gaps and proposes statistical distributions to model the realistic behavior of drivers. To estimate the leading and trailing critical gaps from the field data, maximum likelihood estimation (MLE), a stochastic method has been used. Stochastic methods use probabilistic models to estimate the mean value of critical gaps and the variance of its distribution. The results using MLE and parameters for critical gap distributions are presented in the section on ‘Results’.

All gaps in this paper are time gaps, in seconds. Time gaps are used as they better represent driver behavior compared to distance gaps. Time gaps are also a function of distance gaps and the follower’s speed. The subject vehicle is concerned with sufficient time it needs to make a safe lane change at the current speed. Time gaps are, therefore, a better representation of gaps compared to distance gaps.

The accepted, rejected and critical gaps, in this paper, are analyzed between the subject vehicle and the assumed leader, and the subject vehicle and the assumed follower in the target lane when the leader-follower pairs are observed to be interacting. A vehicle pair is considered to be interacting or car-following when they are separated by 250 feet or less. Herman and Potts (13) observed that for separations greater than 61 m (200 ft), car-following is negligible on driver's behavior. Additionally, Bham and Benekohal (14) observed that drivers are in car-following when separated by 250 feet or less. A distance of 250 feet is, therefore, considered to be appropriate to determine a pair of vehicles to be interacting. Additionally, to quantify interaction between the vehicles in terms of time gaps, the leading and trailing time gaps which are less than or equal to five seconds have been used in the data analysis.

Accepted Gaps
Leading and trailing accepted time gaps for different maneuvers under uncongested and congested traffic conditions were observed from the field data. Histograms of accepted leading and trailing time gaps for mandatory lane change maneuvers were plotted and statistical distributions were fitted to the data. The gamma and lognormal distributions were fitted to the data and Kolmogorov-Smirnov (KS) test was used to test the goodness of fit for both the distributions. For the KS test, the distribution function for the field data, F(x), and model, G(x), were calculated. The KS values were then calculated and compared with the values from the KS Table. The null and alternative hypotheses used were \( H_0: F(x) = G(x) \) versus \( H_1: F(x) \neq G(x) \) for all x. \( H_0 \) was rejected at level \( \alpha \), if \( \text{KS}_{\text{Observed}} > \text{KS}_{\text{Table}} \). The distributions were tested at the 0.05 level of significance and the results are discussed under ‘Results’.
Critical Gaps

Several methods are available for estimating critical gaps for unsignalized intersections. However, some of these methods are not applicable for freeway conditions as these methods require use of lag gaps and major street volumes. Lag gap is the first gap that is available for the minor street vehicle after it comes to a complete stop to merge onto a major street. It is not possible to measure lags on freeways as the complete stop situation does not occur before a lane change.

For consistent driver behavior, i.e. assuming that the rejected gaps are smaller than the accepted gaps, in general, the relationship between accepted, critical, and rejected gaps can be expressed as:

\[ g_r \leq g_c \leq g_a \]  

where,

- \( g_r \) = rejected gap,
- \( g_c \) = critical gap, and
- \( g_a \) = accepted gap.

Due to the stochastic nature of driver behavior, the above relationship, however, does not hold for inconsistent drivers for all cases of gap acceptance and rejection (12).

Gaps not accepted by the subject vehicle five seconds prior to a lane change were considered as rejected gaps in this paper. Five seconds were chosen as largest rejected gaps less than the accepted gaps, indicated as LRLA, were considered. A previous paper (12) analyzed the largest rejected gap prior to a lane change which accounted for most reasons, if not all, for gap rejection and used 10 seconds of rejected gaps. The critical gaps were estimated using LRLA in this paper. This was considered to be the most appropriate way to represent consistent driver gap rejection behavior. This can be expressed as:

\[ g_{lr} = \text{Max}(g_{r1}, \ldots g_{rn}) < g_a \]  

where,

- \( g_{lr} \) = largest rejected gap less than the accepted gap, LRLA,
- \( g_{r1} \) = rejected gap less than the accepted gap, one second prior to the lane change,
- \( g_{rn} \) = rejected gap less than the accepted gap, \( n^{th} \) second prior to the lane change,
- \( g_a \) = accepted gap.

In case, all of the five rejected gaps were higher than the accepted gap, to analyze consistent driver behavior, the lane change maneuver was excluded from the data for estimating the critical gaps.

In the following, a stochastic method applicable to freeway conditions, used for estimating the critical time gaps using accepted and rejected gaps using the LRLA criteria is described. This is carried out to represent the consistent lane change behavior of drivers that adheres to the inequality in Equation 1. Details of values of critical gaps presented in this paper are analyzed in the results section.
Estimating Critical Gaps

Critical gaps can be estimated using the maximum likelihood estimation method for drivers making mandatory lane changes. Analyses of results from this method are presented in the next section.

For a freeway, it is assumed that observations of $n$ drivers, $i = 1, \ldots, n$, have been made, the observations for each include the rejected gap, $b_i$ and the accepted gap, $a_i$. The likelihood $L$ of these observations, as derived by Miller (3) and by Miller and Pretty (15), is

$$L = \text{constant} + \sum_{i=1}^{n} \{ F(b_i) - F(a_i) \}$$  \hspace{1cm} (3)

where $F$ is the cumulative distribution of gaps among drivers who have been observed. $F(a_i)$ is the probability that a gap $a_i$ will be accepted and $F(b_i)$ is the probability that a gap $b_i$ will be rejected by a randomly selected driver. The likelihood that the drivers behaved as they did—as described by the observations—should be as large as possible; that is, one wishes to find the parameters of the distribution $F$. This likelihood is obtained by setting:

$$\frac{\partial L}{\partial p_i} = \left[ \frac{\partial F(b_i)}{\partial p_i} - \frac{\partial F(a_i)}{\partial p_i} \right] = 0, \quad i = 1, \ldots, k$$  \hspace{1cm} (4)

for a distribution with parameters, $p_i$, and then solving the resulting equation system.

As critical gap was found to be a function of accepted and rejected gaps, which were determined to be gamma distributed from the field data, explained next, therefore, a gamma distribution for critical gaps was assumed with a shape parameter $\alpha$ and a scale parameter $\beta$. The cumulative distribution function (CDF) is given as (16):

$$F(X; \alpha, \beta) = \frac{1}{\Gamma(\alpha)} \int_0^X t^{\alpha-1} e^{-t/\beta} \frac{dt}{\beta^\alpha}$$  \hspace{1cm} (5)

where,

- $X = \text{continuous random variable (time gap)}$, and
- $\Gamma(\alpha) = \text{gamma function} = \int_0^\infty t^{\alpha-1} e^{-t} dt$ for all $\alpha > 0$.  \hspace{1cm} (6)

The partial derivatives of the likelihood $L$ with respect to $\alpha$ and $\beta$, and the partial derivatives of CDF of accepted gaps and rejected gaps were determined. The maximum likelihood estimates of the parameters of the gamma distribution $\alpha$ and $\beta$ were determined by setting the derivatives equal to zero and solving iteratively. The mean critical gaps were determined as $\alpha \beta$ and the variance of gamma distributed critical gaps were determined as $\alpha \beta^2$. The section on results describes the values of critical gaps thereby obtained.

FIELD DATA SETS AND DATA REDUCTION

This paper analyzes two 30-minute data sets collected by the NGSIM project (1) on I-80 in Emeryville, California. Figure 1a shows the site for the first data set which was collected from 2:35 to 3:05 p.m. in April, 2004 and represents uncongested traffic conditions. The data set consist of detailed vehicle trajectories collected at $1/15^{th}$ of a second for a half-mile...
section. The longitudinal and lateral coordinates of vehicles were also provided in the data set. Figure 1b shows the site for the second data set which consists of two sub-data sets of 15 minutes each, collected from 5:00 to 5:30 p.m. in April, 2005 at 1/10th of a second. In this paper, the first data set is referred to as the uncongested data and the second data set is referred to as the congested data.

Traffic flow ranges from 600 to 1940 veh/hr/ln in the data sets and as traffic breaks down in the congested data set, the flows are observed in the lower part of the flow-density curve. Traffic density ranges from 21 to 135 veh/mi.

Mandatory lane change maneuvers, analyzed in this paper, are considered as driver maneuvers from the adjacent lane to the shoulder lane to exit the highway from the off-ramp and driver maneuvers from the shoulder lane to the adjacent lane to merge with the main highway lanes.

ANALYSIS OF RESULTS
Analyses of accepted, rejected and critical gaps are presented in this section. Comparisons of mean values and median values of leading and trailing accepted, rejected and critical gaps for uncongested & congested traffic conditions have been carried out. Further, analyses of consistent and inconsistent driver behaviors have been presented.

Firstly, the lane change movements were analyzed. These included both the consistent and inconsistent gap acceptance behavior of drivers. In Table 2, a total of 976 lane changes were analyzed out of which 565 (58%) occurred during uncongested and 411 (42%) during congested traffic conditions. Secondly, the lane changes representing consistent driver behavior were separated out and analyzed. Further, critical gaps were estimated for the consistent driver behavior dataset only. Table 3 represents consistent driver behavior, out of a total of 686 lane changes (70%), 327 (48%) occurred during uncongested and 359 (52%) during congested traffic conditions.

Accepted Gaps
Tables 2 and 3 present the basic statistics of accepted gaps and the different categories of rejected gaps for mandatory lane change maneuvers. Table 2 presents accepted gaps, mean rejected gaps and median rejected gaps. Seventy percent of the data in the table is based on consistent driver behavior and the remaining on inconsistent driver behavior. Table 3 is a subset of Table 2 and presents statistics on consistent driver behavior data only, mainly on accepted gaps and largest rejected gaps less than the accepted gaps (LRLA).

In Table 2, the mean values of accepted time gaps for both the NGSIM data sets were found to be greater than the median values of accepted time gaps. The mean value of mean accepted gaps was found to be 1.34 secs and the mean value of the median accepted gaps was found to be 1.13 secs. The higher values of mean accepted gaps compared to the median values indicate that few drivers (cautious drivers) accepted larger gaps compared to the other drivers. Difference was observed between the mean values of mean accepted gaps for different traffic conditions; 1.29 secs for uncongested data versus 1.40 secs for congested data. However, no difference was observed in the median values of accepted time gaps, 1.13 secs for the two data sets.

Table 2 which represents both consistent and inconsistent driver behaviors, it is observed that the mean value of mean accepted gaps is lower than the mean values of mean rejected gaps under both uncongested and congested datasets and for both the leading and
trailing gaps. The mean values of mean accepted gaps are lower than the mean rejected gaps in the range of two to 10% under uncongested conditions, whereas under congested condition mean values of accepted gaps are lower than the mean rejected gaps in the range of 18 to 22%. The difference in the above ranges between uncongested and congested condition is due to the fact that time gaps are rejected at lower speeds under congested conditions compared to uncongested conditions.

Table 3 represents the behavior of consistent drivers i.e. drivers that reject gaps that were smaller than the accepted gaps. For accepted gaps, it is observed that the mean values are slightly higher than the median values in both the datasets except for one case (in bold). The higher values of mean compared to the median indicate that drivers accepting larger gaps may be referred to as conservative drivers. It is also observed that the mean and median values of accepted time gaps are higher under congested conditions as compared to uncongested conditions. The mean values of mean accepted gaps and median accepted gaps under uncongested conditions equal 1.19 secs and 1.16 seconds, respectively, and the mean values of mean accepted gaps and median accepted gaps under congested conditions equal 1.41 secs and 1.31 seconds, respectively. This is because the speeds are lower in congested conditions as compared to uncongested conditions, therefore, the time gaps are higher.

In Table 3, for accepted gaps, the mean values of trailing and leading accepted gaps were evaluated. The mean value of trailing gaps, 1.35 secs was found to be higher than the mean value of leading gaps, 1.25 secs considering both the datasets. Drivers were found to be more cautious in accepting the trailing gaps as compared to the leading gaps. This is due to the reason that the trailing gaps are harder to perceive compared to leading gaps. Mean value of leadings gaps, 1.35 secs is higher under congested conditions as compared to uncongested conditions, 1.16 secs. Similarly, mean value of trailing gaps, 1.48 secs is higher under congested conditions as compared to uncongested conditions, 1.23 secs. Time gaps for congested conditions are lower compared to uncongested conditions as time gaps are determined using speeds and the average speeds of the vehicles under congested conditions are lower as compared to uncongested conditions.

In Table 3, the mean and median values of accepted for trailing time gaps are higher as compared to leading time gaps for both uncongested and congested conditions and for both adjacent lane to shoulder lane and shoulder lane to adjacent lane maneuvers. This is due to the fact that the drivers have a better perception of leading gaps as compared to trailing gaps. For trailing gaps, drivers look for the assumed follower either in the rear view mirror or over the shoulder, therefore, the perception of leading gaps is better than the trailing gaps.

Further, from Table 3 it is observed from the mean and median values of accepted gaps under both the congested and uncongested traffic conditions and for both the leading and trailing gaps, drivers accept larger gaps for making a lane change from adjacent to the shoulder lane (towards the right) and exiting the highway as compared to the drivers making a lane change from the shoulder lane to the adjacent lane (towards the left) and entering the main lanes of the highway.

Analysis of Table 3, for consistent driver behavior, also shows that the mean values of accepted gaps are greater by seven to 26% greater compared to the mean values of LRLA when mean values are individually compared for each column. Similar values are observed for median values of accepted gaps when compared to the median values of LRLA when compared for each column.
For consistent driver behavior, it is observed that the difference in accepted gaps is similar under uncongested and congested datasets but difference exists between leading and trailing gaps. For leading gaps, for both the traffic conditions, the difference ranges from seven to 10% in leading gaps whereas the difference for trailing gaps ranges from 28 to 34%. This shows that for consistent drivers, there is difference in accepted gaps between leading and trailing gaps and a difference in the values between the accepted and rejected gaps between the leading and trailing gaps. For trailing gaps, drivers tend to reject larger values of gaps as compared to leading gaps. This is consistent with the findings that the accepted gaps are larger for the trailing gaps as compared to the leading gaps.

Moreover, the mean values of leading and trailing accepted gaps are higher for the adjacent to shoulder lane maneuvers compared to the shoulder lane to the adjacent lane maneuvers in both the datasets. This is due to the fact that the frequency of lane changes is higher from the shoulder lane to the adjacent lane (777) as compared to from the adjacent lane to the shoulder lane (199). Due to the higher frequency, drivers may take higher risks and accept smaller gaps. In Table 3, the statistics of accepted gaps shows that the trends are similar to Table 2.

From analyzing Tables 2 and 3, it can be concluded from the mean values of the accepted gaps and mean values of mean, median and LRLA rejected gaps that around 30% of drivers showed inconsistent behavior. To represent the critical gap values for consistent driver behavior in the datasets, values in Table 3 were used to estimate the critical gap values and their distributions which are discussed in the section on ‘Critical Gaps’.

Table 4 shows values of gamma distribution parameters, divided by congested and uncongested traffic conditions, by maneuver, and leading and trailing gaps. The values are specified for the shape parameter $\alpha$, which ranges from 1.65 to 2.42 and scale parameter, $\beta$ which ranges from 0.54 to 0.74. Table 4 also presents the values of KS statistics, $KS_{\text{Observed}}$ for the field data and $KS_{\text{Table}}$ for gamma distribution and the results of hypothesis testing. The hypothesis testing proves that gamma distribution and the parameters found can be used for modeling gap acceptance behavior of drivers in microscopic traffic simulation models under uncongested and congested traffic conditions for mandatory lane changes.

Figure 2 shows the frequency distribution of accepted gaps, subset of accepted gaps (using LRLA criteria) indicated by accepted gaps for LRLA, and rejected gaps (mean, median and LRLA) for leading (Figure 2a) and trailing (Figure 2b) time gaps between the adjacent lane and the shoulder lane for uncongested traffic conditions. Figure 3 and Figure 4 presents the cumulative percentage of the above distributions. Time gap data is grouped into 0.5 second intervals. Gamma and lognormal distributions were attempted to fit the accepted gap data for LRLA criteria, and KS test was used to check the goodness of fit for the distributions. It is observed that the null hypothesis ($H_0$) was rejected for lognormal distribution for most of the maneuvers. The gamma distribution fitted well for the accepted gaps (for LRLA) for all the maneuvers.

**Rejected Gaps**

Table 2 presents the statistics of mean and median rejected gaps. It is observed that the mean value of mean rejected gaps, 1.55 secs and mean value of median rejected gaps, 1.39 secs are greater than the mean value of mean accepted gaps, 1.34 secs and mean value of median accepted gaps, 1.13 secs. This can also been observed in Figure 3 (a) and Figure 4 (a) that the cumulative frequency of mean rejected gaps and the median rejected gaps is towards the right
of the cumulative frequency distribution of the corresponding accepted gaps. Equation 1, therefore, does not hold for all mandatory lane change maneuvers as 30% of drivers in the NGSIM data violate the inequality when the mean and median values of rejected gaps are evaluated. If the gaps rejected by the drivers are higher than the accepted gaps then the gaps were rejected because of several reasons and these reasons were other than collision avoidance. By general definition as indicated in Equation 1, the rejected gaps should be smaller than the accepted gaps for consistent driver behavior; therefore, the mean and median rejected gaps presented in Table 2 should not be utilized for the estimation of critical gap distribution. To achieve realistic gap rejection behavior of drivers for estimation of critical gaps, the rejected gaps should be less than the accepted gaps. The largest rejected gap less than the accepted gap (LRLA), therefore, represents the realistic case of gap rejection for consistent drivers.

Table 3 presents the statistics of mean and median of LRLA gaps. It is observed that the mean value of mean rejected gaps, 1.09 secs and mean value of median rejected gaps, 1.05 secs are less than the mean value of mean accepted gaps, 1.30 secs and mean value of median accepted gaps, 1.24 secs. This is contrary to the observation in Table 2 and explained in the above paragraph. Table 3, therefore, presents the results for consistent drivers which hold the inequalities in Equations 1 and 2.

In Table 3, for rejected gaps, it is observed that the mean values of LRLA for trailing gaps are shorter than the leading gaps for all pairs of leading and trailing gaps. This is due to the reason that leading gaps are easier to perceive compared to trailing gaps, therefore, a higher number of shorter trailing gaps are rejected. It is also observed that the difference between the accepted and LRLA is higher for trailing gaps as compared to the leading gaps, and that the difference between the accepted and LRLA is larger for congested conditions as compared to the uncongested conditions. This shows that drivers are extra cautious as the vehicles are close to each other under congested conditions.

In Table 3, for trailing gaps it is observed that the median values of LRLA are larger than the mean values except for one case (in bold), whereas for leading gaps the median values of LRLA are shorter than the mean values. This indicates that for trailing gaps few drivers reject very smaller values of gaps as compared to the driver population and for leading gaps few drivers reject very large values of gaps as compared to the driver population.

Similar to the results in Table 2, it is observed from Table 3 that the drivers reject larger gaps when making a lane change from the adjacent lane to the shoulder lane (towards right) as compared to when making a lane change from the shoulder lane to the adjacent lane (towards left). This shows the gap rejection behavior of drivers and that they have a better perception of gaps when making a lane change towards the left as compared to making a lane change towards the right.

From Figure 3 (b) and Figure 4 (b), it can be seen that the cumulative frequency distribution of LRLA is towards the left of the corresponding accepted gaps. These figures also clearly indicate the difference in accepted gaps for trailing and leading gaps; that trailing gaps are larger than the leading gaps.

**Critical Gaps**

Critical gaps were estimated using LRLA and the corresponding accepted gaps presented in Table 3 that are consistent with Equation 1. Table 4 presents the values of critical time gaps.
The results indicate that the values of trailing critical gaps are higher than the values of leading critical gaps in both the uncongested and congested traffic conditions. This result is similar to results observed for accepted gaps which shows that the drivers making a lane change prefer larger trailing gaps than the leading gaps. Similarly, critical time gaps are determined to be greater under congested conditions as compared to uncongested conditions. This finding may also be because under congested conditions, presented in Figure 1(b), data was available until 355 feet before the off-ramp. As the drivers approach the off-ramp when traveling on the shoulder lane, the accepted gaps got shorter as the drivers approached the lane drop.

From Table 4, it is observed that the trailing and leading critical gaps are 22% and 15% greater, respectively, during congested conditions compared to the uncongested conditions. It is also observed that under uncongested and congested conditions, the average value of critical gaps from the adjacent lane to the shoulder lane is about 16 to 18% higher than the average value from the shoulder lane to the adjacent lane. This shows that drivers merging from the on-ramp and merging to the adjacent lane accept shorter gaps and do not reject larger gaps in comparison to the drivers exiting from the off-ramp.

Critical gaps, based on the distribution of accepted and rejected gaps were assumed to be gamma distributed as critical gaps were determined using values of accepted and LRLA gaps. Table 6 presents the parameters of critical gap distribution estimated using the methodology presented under estimation of critical gaps. These parameter values can be used to generate gamma distributed critical gaps which can be assigned to drivers in microscopic simulation models.

In a previous study (12) for estimating critical gaps using the largest rejected gaps, it was found that the largest rejected gaps are greater than the accepted gaps, and as a result, values of critical gaps estimated using the largest rejected gaps lie between the mean value of the accepted gaps and the mean value of the largest rejected gaps. The critical gaps estimated using the largest rejected gaps and the accepted gaps can be used to determine the threshold of the largest rejected gaps. The threshold of the largest reject gap represents the value above which the drivers will not accept a gap. To understand the utilization of the critical gaps estimated using LRLA and the critical gaps estimated using the largest rejected gaps (LR) (12), the mean values of different gaps is presented below:

\[\begin{align*}
\text{LRLA} & = \text{Mean value of largest rejected gaps less than the accepted gaps = 1.09 seconds (from Table 3)}, \\
C_{\text{LRLA}} & = \text{Critical gap estimated using LRLA, (LRLA < C_{\text{LRLA}} < A) = 1.19 seconds (from Table 5)}, \\
A & = \text{Mean value of accepted gaps = 1.30 seconds (from Table 3)}, \\
C_{\text{LR}} & = \text{Critical gap estimated using the largest rejected gaps (LR > C_{\text{LR}} > A) = 1.72 seconds (12), and} \\
LR & = \text{Mean of largest rejected gaps = 4.00 seconds (derived from data set (1))}
\end{align*}\]

The mean value of largest rejected gaps, 4.00 secs is much larger than the mean value of accepted gaps 1.3 secs. This represents gap rejection due to reasons other than collision avoidance such as the location was not the intended position of lane change, etc. The largest rejected gap less than the accepted gap represents rejected gaps because of unsafe lane change conditions such that can cause collision if lane change is performed. The mean value
of accepted gaps is greater than mean value of LRLA and provides a safe lane change when the lane change is made. Critical gaps based on the largest rejected and accepted gaps represent the threshold of gap rejection which shows that drivers reject gaps which are larger than the accepted gap due to reasons other than safe lane change maneuvers. If the available gap is greater than $C_{LRLA}$, then it is acceptable for a safe lane change maneuver.

$C_{LR}$ can be used in gap rejection so that an available gap that is larger than $C_{LR}$, the lane change can be delayed and the driver does not make a lane change as soon as it finds the first available gap. This is important because if only $C_{LRLA}$ or collision avoidance conditions are used in gap acceptance models for mandatory lane changes then the vehicles will make lane changes the moment these conditions are met. This is not a realistic representation of lane changes in terms of intended location of lane change. By using $C_{LR}$, a threshold of largest rejected gap can be induced in gap acceptance models. This will add stochasticity in driver lane change behavior by using such gap acceptance models. Using $C_{LR}$ along with $C_{LRLA}$ in gap acceptance algorithms will provide realistic representation of both gap acceptance and gap rejection behavior of the drivers.

**Distribution Parameters for Critical Gaps**

Table 6 presents the mean and variance of critical gaps determined using the gamma distribution. Earlier studies assumed exponential (6), lognormal (7), and normal (3) distributions for the critical gaps, however, the data analyzed for this paper and in a previous paper (12), critical gaps were found to be gamma distributed based on accepted and rejected gaps.

From Table 6, the mean leading and trailing critical gaps for the combined datasets (uncongested and congested) and for the both the lane change maneuvers combined, were found to be 1.17 and 1.26 seconds, and the variance to be 0.82 and 0.87 seconds$^2$, respectively. These values of distribution parameters may be used for generating gamma distributed critical gaps in different traffic conditions and for different maneuvers in microscopic simulation models. Figure 5 shows the CDF of gamma distributed critical gaps determined, accepted gaps and LRLA for both the leading and trailing gaps for the adjacent lane to the shoulder lane maneuver.

**CONCLUSIONS AND RECOMMENDATIONS**

This research proposes critical gaps and its distribution parameters which can be used to replicate drive gap acceptance behavior in traffic simulation models. Mandatory lane change maneuvers observed in the field data do not always follow collision avoidance conditions and gaps are rejected because of reasons that than collision avoidance. Critical gaps, therefore, are proposed as a driver behavior characteristic that represents realistic driver behavior.

Gamma distribution is proposed for accepted time gaps as it fits well for both leading and trailing accepted time gaps for both the uncongested and congested traffic flow conditions. It can be concluded that gamma distribution represents the gap acceptance behavior of drivers during a lane change and can be used in microscopic traffic simulation models. Lognormal distribution is not recommended for accepted gaps as the null hypothesis is rejected for most of the maneuvers, and therefore, the distribution fails to represent realistic driver gap acceptance behavior.
From the mean values of estimated critical time gaps, trailing critical gaps were found to be higher than the leading critical gaps. This result signifies that the drivers are more sensitive to trailing critical gaps compared to leading critical gaps. Results also showed that for mandatory lane changes, critical gaps are larger for congested conditions compared to uncongested conditions.

Results obtained from the distribution of accepted time gaps and estimated critical time gaps can be used in traffic simulation models to generate values of accepted gaps and critical gaps for mandatory lane changes. Critical gaps estimated using the largest rejected gaps that are less than the accepted gaps can be used along with the critical gaps estimated using largest rejected gaps to get realistic representation of driver gap acceptance and gap rejection behavior. This procedure can also be used in existing lane change models that utilize the collision avoidance check for allowing a lane change. Also, leading and trailing accepted and rejected gap values from the field data may be used for calibrating traffic simulation models. Critical gap values may also be used to identify the point where drivers start to make a lane change and traffic signs can be provided on the freeway. Further study is recommended to estimate critical gaps for discretionary lane changes and multiple lane changes. This paper concentrated on the consistent behavior of drivers, further study is needed on the inconsistent behavior of drivers. Further research can be carried out on the development of a simulation model based on critical gaps, gap rejection and gap acceptance based on field data and using the concepts of largest rejected gaps less than the accepted gaps and the largest rejected gaps.

ACKNOWLEDGEMENTS
We acknowledge the NGSIM data sets provided by Cambridge Systematics. The support of the Civil Engineering Department at the Missouri University of Science and Technology and the research grant from the University of Missouri Research Board is also appreciated.

REFERENCES
11) Hidas, P. A Microscopic Study of Lane Changing Behavior. 24th CAITR Conference, the University of South Wales, December, 2002.
Table 1. Terminologies used in the paper

- Mandatory lane changes: the essential lane changes that drivers make to exit from the off-ramp, enter main lanes from the on-ramp, or merge to the adjacent lane to avoid lane drop.
- Subject vehicle: the vehicle that intents to make a lane change.
- Assumed leader: the immediate leader to the subject vehicle in the target lane.
- Assumed follower: the immediate follower to the subject vehicle in the target lane.
- Leading time gaps: calculated as the distance between the rear bumper of the assumed leader and the front bumper of the subject vehicle over the speed of the subject vehicle.
- Trailing time gaps: calculated as the distance between the rear bumper of the subject vehicle to the front bumper of the assumed follower over the speed of the assumed follower.
- Shoulder lane: the rightmost lane.
- Adjacent lane: the lane next to the shoulder lane (Figure 1).
- Leading accepted gap: the time gap between the subject vehicle and the assumed leader at a time frame when the center of the front bumper of the subject vehicle just crosses the lane marking.
- Trailing accepted gap: the time gap between the subject vehicle and the assumed follower at a time step when the center of the front bumper of the subject vehicle just crosses the lane marking.
- Rejected leading gap: the time gap between the subject vehicle and the assumed leader between a lane change and five seconds prior to a lane change.
- Rejected trailing gap: the time gap between the subject vehicle and the assumed follower between a lane change and five seconds prior to a lane change. In this paper, rejected gaps mean the largest rejected gap less than the accepted gap or LRLA, unless otherwise stated.
- Gaps are classified further as follows:
  - Rejected Gaps
    - Mean rejected gaps – Mean of five rejected gaps prior to a lane change.
    - Median rejected gaps – Median of five rejected gaps prior to a lane change.
    - Largest rejected gap less than the accepted gap (LRLA) - Largest value among the five rejected gaps which is less than the accepted gap.
    - Largest rejected gap – Largest value of rejected gap.
  - Critical Gaps
    \( C_{\text{LRLA}} \) = Critical gap determined using LRLA and accepted gap (A),
    \( (\text{LRLA} < C_{\text{LRLA}} < \text{A}) \),
Table 2. Statistics of Accepted and Rejected Time Gaps (secs)

<table>
<thead>
<tr>
<th>Traffic Condition</th>
<th>Uncongested Data</th>
<th>Congested Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Adjacent to Shoulder Lane</td>
<td>Shoulder to Adjacent Lane</td>
</tr>
<tr>
<td>Statistics</td>
<td>Leading</td>
<td>Trailing</td>
</tr>
<tr>
<td>Accepted</td>
<td>1.21</td>
<td>1.12</td>
</tr>
<tr>
<td>Mean</td>
<td>1.14</td>
<td>1.16</td>
</tr>
<tr>
<td>Median</td>
<td>1.55</td>
<td>1.36</td>
</tr>
<tr>
<td>Rejected</td>
<td>1.31</td>
<td>1.32</td>
</tr>
<tr>
<td>Median</td>
<td>1.34</td>
<td>1.41</td>
</tr>
<tr>
<td>Accepted</td>
<td>0.88</td>
<td>0.93</td>
</tr>
<tr>
<td>Rejected</td>
<td>0.83</td>
<td>0.97</td>
</tr>
<tr>
<td>No. of Observations/Total</td>
<td>109</td>
<td>456</td>
</tr>
</tbody>
</table>

Table 3. Statistics of Accepted and Largest Rejected Gaps less than the Accepted Gaps (LRLA) (secs)

<table>
<thead>
<tr>
<th>Traffic Condition</th>
<th>Uncongested Data</th>
<th>Congested Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Adjacent to Shoulder Lane</td>
<td>Shoulder to Adjacent Lane</td>
</tr>
<tr>
<td>Statistics</td>
<td>Lead</td>
<td>Trail</td>
</tr>
<tr>
<td>Median</td>
<td>1.2</td>
<td>1.27</td>
</tr>
<tr>
<td>Accepted</td>
<td>1.28</td>
<td>1.33</td>
</tr>
<tr>
<td>LRLA</td>
<td>1.19</td>
<td>1.04</td>
</tr>
<tr>
<td>Mean</td>
<td>1.19</td>
<td>1.04</td>
</tr>
<tr>
<td>Accepted</td>
<td>0.88</td>
<td>0.93</td>
</tr>
<tr>
<td>LRLA</td>
<td>0.83</td>
<td>0.97</td>
</tr>
<tr>
<td>No. of Observations/Total</td>
<td>63</td>
<td>264</td>
</tr>
</tbody>
</table>

LRLA = largest rejected gap less than the accepted gap
### Table 4. KS Test for Accepted Time Gaps (for LRLA)

<table>
<thead>
<tr>
<th>Maneuver</th>
<th>Parameters</th>
<th>KS Stat.</th>
<th>Leading</th>
<th>Trailing</th>
<th>Leading</th>
<th>Trailing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>KS Obs.</td>
<td>H₀</td>
<td>KS Obs.</td>
<td>H₀</td>
<td></td>
</tr>
<tr>
<td><strong>Uncongested Data</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adjacent to Shoulder Lane</td>
<td>(\alpha = 2.07) (\beta = 0.62)</td>
<td>0.1680</td>
<td>0.1011</td>
<td>NR</td>
<td>0.1050</td>
<td>NR</td>
</tr>
<tr>
<td>Shoulder to Adjacent Lane</td>
<td>(\alpha = 1.65) (\beta = 0.63)</td>
<td>0.0836</td>
<td>0.0326</td>
<td>NR</td>
<td>0.0618</td>
<td>NR</td>
</tr>
<tr>
<td><strong>Congested Data</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adjacent to Shoulder Lane</td>
<td>(\alpha = 1.98) (\beta = 0.72)</td>
<td>0.1287</td>
<td>0.0921</td>
<td>NR</td>
<td>0.0962</td>
<td>NR</td>
</tr>
<tr>
<td>Shoulder to Adjacent Lane</td>
<td>(\alpha = 1.71) (\beta = 0.73)</td>
<td>0.0975</td>
<td>0.0478</td>
<td>NR</td>
<td>0.0584</td>
<td>NR</td>
</tr>
</tbody>
</table>

Note: NR = Not Rejected

### Table 5. Critical Time Gaps estimated using Accepted Gaps and Largest Rejected Gaps Less than the Accepted Gaps (LRLA)

<table>
<thead>
<tr>
<th>Lane Change Maneuver From – To</th>
<th>Uncongested Data</th>
<th>Congested Data</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Leading</td>
<td>Trailing</td>
<td>Leading</td>
</tr>
<tr>
<td>Adjacent to Shoulder Lane</td>
<td>1.22</td>
<td>1.14</td>
<td>1.35</td>
</tr>
<tr>
<td>Shoulder to Adjacent Lane</td>
<td>0.98</td>
<td>1.02</td>
<td>1.19</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td>1.10</td>
<td>1.08</td>
<td>1.27</td>
</tr>
</tbody>
</table>

### Table 6. Gamma Distribution Parameters for Critical Gaps

<table>
<thead>
<tr>
<th>Maneuver</th>
<th>Uncongested Data</th>
<th>Congested Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Leading</td>
<td>Trailing</td>
</tr>
<tr>
<td></td>
<td>(\alpha) (\beta) (\text{Var}(\alpha \beta^2))</td>
<td>(\alpha) (\beta) (\text{Var}(\alpha \beta^2))</td>
</tr>
<tr>
<td><strong>Uncongested Data</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adjacent to Shoulder Lane</td>
<td>1.54</td>
<td>0.80</td>
</tr>
<tr>
<td>Shoulder to Adjacent Lane</td>
<td>1.49</td>
<td>0.61</td>
</tr>
<tr>
<td><strong>Congested Data</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adjacent to Shoulder Lane</td>
<td>1.88</td>
<td>0.71</td>
</tr>
<tr>
<td>Shoulder to Adjacent Lane</td>
<td>1.78</td>
<td>0.66</td>
</tr>
</tbody>
</table>
a) Uncongested Data with Six Main Lanes, an On-Ramp and an Off-Ramp.

b) Congested Data with Six Main Lanes and an On-Ramp
(Data collected in the dotted area)

Figure 1. Schematic of Datasite at I-80, Emeryville, California
(Note: Figure not to scale)
Figure 2. Frequency Distribution for Accepted, Mean Rejected, Median Rejected, Accepted Gaps for LRLA, and LRLA
a) CDF of Accepted, Mean Rejected and Median Rejected Time Gaps

b) CDF of Accepted (for LRLA) and LRLA Time Gaps

Figure 3. CDF of Leading Time Gaps for Uncongested Conditions from Adjacent Lane to Shoulder Lane
Figure 4. CDF of Trailing Time Gaps for Uncongested Conditions from Adjacent Lane to Shoulder Lane

a) CDF of Accepted, Mean Rejected and Median Rejected Time Gaps

b) CDF of Accepted (for LRLA) and LRLA Time Gaps
Figure 5. CDF of Gamma Distributed Critical Gaps from Maximum Likelihood Estimation