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Analyzing Microscopic Behavior: Driver Mandatory Lane Change Behavior on a Multilane Freeway

by

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16. Abstract

The driver gap acceptance and rejection behavior during mandatory lane changes on a multilane freeway are analyzed in this report. Gaps are accepted or rejected based on comparison with a minimum value generally defined as the critical gaps. Critical gaps are estimated based on the accepted and rejected gaps observed in the field. Driver behavior can be classified as consistent or inconsistent on the basis of gap rejection. For consistent driver behavior, it is assumed that the rejected gaps are shorter than the accepted gaps. This report focuses on the estimation of the critical gaps and its distribution for consistent driver behavior. Critical gap, for consistent driver behavior is defined as the minimum value of gap above which the lane changer does not reject a gap to execute a lane change. Several gaps may be rejected prior to a gap being accepted, therefore, different types of rejected gaps can be utilized to estimate critical gaps. To systematically evaluate rejected gaps and propose the most suitable rejected gaps for use in estimating critical gaps, rejected gaps were analyzed using the mean rejected, median rejected, and the largest rejected gaps less than the accepted gaps (LRLA). To model the consistent gap acceptance behavior of drivers i.e. the rejected gap is less than the accepted gap, LRLA is used in estimating the critical gaps. Accepted, LRLA, and critical gaps were assumed to follow a gamma distribution. The values of critical time gaps were estimated using the maximum likelihood estimation method. This report utilizes data collected by the NGSIM project.

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Driver Mandatory Lane Change Behavior on a Multilane Freeway

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INTRODUCTION

Gap acceptance is an important component of lane change algorithms used in microscopic traffic simulation models. This report presents an analysis of driver gap acceptance and gap rejection behavior using detailed vehicle trajectory data (1) and proposes statistical distributions for accepted and critical time gaps which can be utilized in modeling realistic mandatory lane change behavior of drivers in microscopic simulation models. Common terminologies used in this report are defined in Table 1.

Researchers, in the past, have proposed methods to estimate critical time gaps for unsignalized intersections that are useful in estimating capacity and delay (2, 3). The Highway Capacity Manual (HCM) (4) defines a critical gap for an unsignalized intersection as the minimum time, in seconds, between successive major stream vehicles in which a minor-street vehicle can make a maneuver and enter the major street. To represent other gap acceptance situations such as lane changes on a freeway, merging or diverging on a freeway, or overtaking on a bi-directional road, the more generalized definition of critical gaps is the minimum time between successive vehicles in which the subject vehicle can perform the intended maneuver (5). A minimum value of accepted gaps from a population, however, can not be considered a critical gap as it would only represent the gap acceptance behavior of an aggressive driver and would not represent the minimum acceptable gap of the driver population.

Critical gaps are modeled as random variables and different distributions have been proposed in the literature; Herman and Weiss (6) assumed an exponential distribution, Drew et. al. (7) assumed a log-normal distribution, and Miller (3) assumed a normal distribution. Similarly, lane change models in simulation software such as MITSIM (8), CORSIM (9), VISSIM (10), and SITRAS (11) use critical gaps in different ways. MITSIM assumes the distribution of critical gaps to have a lognormal distribution and takes into account the leading and trailing critical distance gaps. CORSIM defines critical gaps by using risk factors. The risk factor is defined by the rate of deceleration a driver applies to avoid collision if the leader brakes to a complete stop. Risk factors are evaluated for every lane change between the subject vehicle and the assumed leader and between the subject vehicle and the assumed follower. A risk factor is compared to an acceptable risk factor that depends on types of lane changes, drivers, and urgency of lane change. In VISSIM, the psychophysical model presents critical gaps as thresholds that depend on the relative speeds of the subject vehicle and the assumed leader and the subject vehicle and the assumed follower. In SITRAS, a critical gap is defined as the minimum acceptable gap and is equal to the sum of minimum safe constant gap and the product of a constant with relative speed.

Lane change maneuvers may reduce the freeway capacity under different traffic flow conditions. As critical gap distributions are used in lane change models as described above, using the realistic distribution of critical gaps is important in producing the frequency of lane changes close to real world conditions, close replication of driver behavior during lane changes, and freeway capacity estimations. The distribution of critical gaps, therefore, should be determined so it can be used in simulation models to have results that are in close agreement with the field data. Unrealistic results can lead to erroneous results about the characteristics of the facility being simulated.

Furthermore, in past studies for unsignalized intersections (2, 3), driver gap acceptance behavior have been categorized as consistent and inconsistent. Critical gaps, for unsignalized intersections, were estimated assuming consistent driver gap acceptance behavior. Consistent behavior assumes that drivers do not reject gaps that are larger than the accepted gaps. On freeways during an intended lane change, however, drivers reject a gap due to various reasons such as to avoid collision with the assumed leader and the assumed follower in the target lane, have not reached the intended location of lane change, late reaction to traffic signs, etc. The rejected gaps, therefore, may or may not be less than the accepted gaps, contrary to the assumption for consistent drivers at unsignalized intersections. The lane changes on a freeway are also in contrast to gap acceptance at unsignalized intersections where a driver comes to a complete stop before entering the major street. On freeways, a driver seeks a suitable gap while in motion and it is not possible to determine the exact point in space or time when a driver decides to make a lane change. As critical time gaps are a function of accepted and rejected time gaps, and there can be several rejected gaps before a gap is accepted, it is important to consider an appropriate value of a rejected gap to get an estimate of critical gaps that represent realistic driver behavior. To estimate the value of critical gaps, different rejected gaps are evaluated in this report. To analyze the detailed gap rejection behavior, the mean rejected, median rejected and the largest rejected gap less than the accepted gap (LRLA) are used. A paper (12) analyzed the largest rejected gap prior to a lane change which accounted for most reasons, if not all, for gap rejection.

The report is organized as follows: first, the methodology is discussed, second, the field data and data reduction is presented, third the results from the application of the methodology to the field data sets are described and last, conclusions and recommendations are summarized. All gaps referred to, in this report, are time gaps, unless otherwise stated.

METHODOLOGY

This report analyzes NGSIM datasets, described in the next section, to determine the driver leading and trailing accepted and rejected gaps and proposes statistical distributions to model the realistic behavior of drivers. To estimate the leading and trailing critical gaps from the field data, maximum likelihood estimation (MLE), a stochastic method has been used. Stochastic methods use probabilistic models to estimate the mean value of critical gaps and the variance of its distribution. The results using MLE and parameters for critical gap distributions are presented in the section on 'Results'.

Accepted, rejected and critical gaps are represented by time gaps in this report. Time gaps are used as they better represent driver behavior compared to distance gaps. Time gaps are also a function of distance gaps and the follower's speed. The subject vehicle is concerned with sufficient time it needs to make a safe lane change at the current speed. Time gaps are, therefore, a better representation of gaps compared to distance gaps.

The accepted, rejected and critical gaps, in this report, are analyzed between the subject vehicle and the assumed leader, and the subject vehicle and the assumed follower in the target lane when the leader-follower pairs are observed to be interacting. A vehicle pair is considered to be interacting or car-following when they are separated by 250 feet or less. Herman and Potts (13) observed that for separations greater than 61 m (200 ft), car-following is negligible on driver's behavior. Additionally, Bham and Benekohal (14) observed that drivers are in car-following when separated by 250 feet or less. A distance of 250 feet is, therefore, considered to be appropriate to determine a pair of vehicles to be interacting. Additionally, to quantify interaction between the vehicles in terms of time gaps, the leading and trailing time gaps which are less than or equal to five seconds have been used in the data analysis.

Accepted Gaps

Leading and trailing accepted time gaps for different maneuvers under uncongested and congested traffic conditions were observed from the field data. Histograms of accepted leading and trailing time gaps for mandatory lane change maneuvers were plotted and statistical distributions were fitted to the data. The gamma and lognormal distributions were fitted to the data and Kolmogorov-Smirnov (KS) test was used to test the goodness of fit for both the distributions. For the KS test, the distribution function for the field data, F(x), and model, G(x), were calculated. The KS values were then calculated and compared with the values from the KS Table. The null and alternative hypotheses used were H_0: F(x) = G(x) versus H_1: F(x) ≠ G(x) for all x. H_0 was rejected at level α, if $KS_{Obserevd} > KS_{Table}$. The distributions were tested at the 0.05 level of significance and the results are discussed under 'Results'.

Critical Gaps

Several methods are available for estimating critical gaps for unsignalized intersections. However, some of these methods are not applicable for freeway conditions as these methods require use of lag gaps and major street volumes. Lag gap is the first gap that is available for the minor street vehicle after it comes to a complete stop to merge on to a major street. It is not possible to measure lags on freeways as the complete stop situation does not occur before a lane change.

For consistent driver behavior, i.e. assuming that the rejected gaps are smaller than the accepted gaps, in general, the relationship between accepted, critical, and rejected gaps can be expressed as:

$$g_r < g_c \leq g_a \tag{1}$$

where,

g_r = rejected gap,
g_c = critical gap, and
g_a = accepted gap.

All gaps in this report are in seconds. Due to the stochastic nature of driver behavior, the above relationship, however, does not hold for inconsistent drivers for all cases of gap acceptance and rejection (12).

Gaps not accepted by the subject vehicle five seconds prior to a lane change were considered as rejected gaps in this report. The critical gaps were estimated using the largest rejected gaps less than the accepted gaps indicated by LRLA in this report. This was considered to be the most appropriate way to represent consistent driver gap rejection behavior. This can be expressed as:

$$g_{lr} = Max(g_{r1},...g_{rn}) < g_a \tag{2}$$

where,

g_{lr} = largest rejected gap less than the accepted gap,
g_{r1} = rejected gap less than the accepted gap, one second prior to the lane change,
g_{rn} = rejected gap less than the accepted gap, n^{th} second prior to the lane change, and
g_a = accepted gap.

In case, all of the five rejected gaps were higher than the accepted gap, to analyze consistent driver behavior, the lane change maneuver was excluded from the data for estimating the critical gaps.

In the following, a stochastic method applicable to freeway conditions, used for estimating the critical time gaps using accepted and rejected gaps using the LRLA criteria is described. This is carried out to represent the consistent lane change behavior of drivers that adheres to the inequality in Equation 1. Details of values of critical gaps presented in this report are discussed under 'Results'.

Maximum Likelihood Estimation

Critical gap is modeled as a random variable in stochastic methods. Maximum likelihood estimation is used, in this report, to estimate leading and trailing critical time gaps for mandatory lane changes. Results from this method are presented under 'Results'.

Maximum likelihood estimation (3) treats the gap acceptance process as a binary function with accepted and rejected gaps represented as one and zero, respectively. Miller (3) proposed a model in which the probability that the gap rejected by a driver is in the range (a$\pm\frac{1}{2}\delta$a), where 'a' is the LRLA gap by a driver, and the gap accepted will be in the range (b$\pm\frac{1}{2}\delta$b), where 'b' is the gap accepted by a driver. The probabilities of accepted and rejected gaps are expressed as (3):

$$\delta a\delta b\int_a^b f(x)\sum_{r=1}^{\infty} rg(a)\left[G(a)^{r-1}\right]g(b)dx,\ \text{for a} \neq 0, \tag{3}$$

where,

x	= critical gap of a driver,	
f ()	= p.d.f. of critical gaps,	
r	= number of gaps rejected by a person,	
g ()	= p.d.f. of gaps in the traffic stream, and	
G ()	= c.d.f of gaps in the traffic stream.	

Performing the summation and integration in Equation 3 gives:

$$\delta a\delta b\{F(b)-F(a)\}\frac{g(a)g(b)}{[1-G(a)]^2} \tag{4}$$

where,

F () = c.d.f. of critical gaps.

As δa and δb are small and therefore dropped, we get the "likelihood" of the data pair (a, b). If a sample of 'n' drivers with values of (a_i, b_i) are considered then the likelihood of this sample data is:

$$\prod_{i=1}^{n}\{F(b_i)-F(a_i)\}\prod_{i=1}^{n}\frac{g(a_i)g(b_i)}{[1-G(a_i)]^2} \tag{5}$$

The method of maximum likelihood determines the values of the parameters which maximize the likelihood function 'L'. The parameters of gap acceptance distribution occur only in the left term (product) in Equation 5 and so the second product is a constant for any given dataset. The likelihood function 'L' for the critical gap is expressed as a function of accepted and rejected gaps as:

$$\text{L} = \text{constant} + \sum_{i=1}^{n}\{F(b_i) - F(a_i)\} \tag{6}$$

As critical gap is a function of accepted and rejected gaps which is determined to be gamma distributed from the field data explained in the next section, therefore, to maximize the likelihood of a critical gap, a gamma distribution for critical gaps is assumed with a shape parameter (α) and a scale parameter (β). The cumulative distribution function (CDF) of X ~ Gamma (α, β) is given as (15):

$$\text{F}(\text{X}; \alpha, \beta) = \int_0^x \frac{t^{\alpha-1}e^{-t/\beta}}{\beta^{\alpha}\Gamma(\alpha)}dt \tag{7}$$

where,

X = continuous random variable (in this case X is 'time gap'), and

$$\Gamma(\alpha) = \text{gamma function} = \int_0^{\infty} t^{\alpha-1}e^{-t}dt \quad \text{for all } \alpha > 0. \tag{8}$$

The partial derivates with respect to parameters α and β were determined using a computer program written by the authors. The mean is determined as αβ and the variance of gamma distributed critical gaps is determined as $\alpha\beta^2$. Partial derivatives of the likelihood 'L' with respect to α and β are as follows:

$$\frac{\partial L}{\partial \alpha} = \left[\frac{\partial F(b_i)}{\partial \alpha} - \frac{\partial F(a_i)}{\partial \alpha}\right] \tag{9}$$

$$\frac{\partial L}{\partial \beta} = \left[\frac{\partial F(b_i)}{\partial \beta} - \frac{\partial F(a_i)}{\partial \beta}\right] \tag{10}$$

Following partial derivatives of CDF of accepted (b) and rejected gaps (a) with respect to α and β were calculated as:

$$\frac{\partial[\text{F}(\text{b}_\text{i})]}{\partial \alpha} = \frac{\partial\left[\int_0^{\text{b}} \frac{\text{t}^{\alpha-1}\text{e}^{-\text{t}/\beta}}{\beta^{\alpha}\Gamma(\alpha)}\text{dt}\right]}{\partial \alpha} \tag{11}$$

$$\frac{\partial[\text{F}(\text{a}_\text{i})]}{\partial \alpha} = \frac{\partial\left[\int_0^{\text{a}} \frac{\text{t}^{\alpha-1}\text{e}^{-\text{t}/\beta}}{\beta^{\alpha}\Gamma(\alpha)}\text{dt}\right]}{\partial \alpha} \tag{12}$$

$$\frac{\partial[F(b_i)]}{\partial\beta} = \frac{\partial\left[\int_0^b \frac{t^{\alpha-1}e^{-t/\beta}}{\beta^\alpha\Gamma(\alpha)}dt\right]}{\partial\beta} \tag{13}$$

$$\frac{\partial[F(a_i)]}{\partial\beta} = \frac{\partial\left[\int_0^a \frac{t^{\alpha-1}e^{-t/\beta}}{\beta^\alpha\Gamma(\alpha)}dt\right]}{\partial\beta} \tag{14}$$

The sum of squares of Equation 9 and 10 were maximized using the computer program as mentioned previously and the maximum likelihood estimates of the parameters of the gamma distribution (α and β) were determined. Table 6 presents the values of critical gap parameters following the gamma distribution. Figure 5 presents the gamma distribution of accepted, critical and LRLA rejected gaps.

FIELD DATA SETS AND DATA REDUCTION

This report presents analyses of two 30-minute data sets collected by the NGSIM project (1) on I-80 in Emeryville, California. Figure 1a shows the site for the first data set which was collected from 2:35 to 3:05 p.m. in April, 2004 and represents uncongested traffic conditions. The data set consist of detailed vehicle trajectories collected at $1/15^{th}$ of a second for a half-mile section. The longitudinal and lateral coordinates of vehicles were also provided in the data set. Figure 1b shows the site for the second data set which consists of two sub-data sets of 15 minutes each collected from 5:00 to 5:30 p.m. in April, 2005 at $1/10^{th}$ of a second. In this report, the first data set is referred to as the uncongested data and the second data set is referred to as the congested data.

Traffic flow ranges from 600 to 1940 veh/hr/ln in the data sets and as traffic breaks down in the congested data set, the flows are observed in the lower part of the flow-density curve. Traffic density ranges from 21 to 135 veh/mi in the data sets.

Mandatory lane change maneuvers, discussed in this report, are considered as driver maneuvers from the adjacent lane to the shoulder lane to exit the highway from the off-ramp and driver maneuvers from the shoulder lane to the adjacent lane to merge with the main highway lanes**.**

RESULTS AND DISCUSSION

Driver behavior for accepted, rejected and critical gaps has been presented in this section. Comparisons of mean values and median values of leading and trailing accepted, rejected and critical gaps for uncongested & congested traffic conditions as well for different maneuvers have been carried out. Further, analyses of consistent and inconsistent driver behaviors have been carried out.

Firstly, the lane changes movements were analyzed. These included both the consistent and inconsistent driver behavior of gap acceptance. In Table 2, a total of 976 lane changes were analyzed out of which 565 (58%) were for uncongested and 411 (42%) were for congested traffic conditions. Secondly, lane changes representing consistent driver behavior were separated out and analyzed. Further, critical gaps were estimated for the consistent driver behavior dataset. In Table 3, a total of 686 lane changes (70%) were observed for consistent driver behavior out of which 327 (48%) were for uncongested and 359 (52%) were for congested traffic conditions.

Accepted Gaps

Tables 2 and 3 show the basic statistics of accepted gaps and the different categories of rejected gaps for mandatory lane change maneuvers. Table 2 presents' accepted gaps, mean rejected gaps and median rejected gaps. Seventy percent of the data in the table is based on consistent driver behavior and the remaining on inconsistent driver behavior. Table 3 is a subset of data presented in Table 2 and represents consistent driver behavior data. Table 3 presents statistics of accepted gaps with those of largest rejected gaps less than the accepted gaps (LRLA).

In Table 2, the mean values of accepted time gaps for both the NGSIM data sets were found to be greater than the median values of accepted time gaps. The mean value of accepted gaps was found to be 1.34 secs and the mean value of the median accepted gaps was found to be 1.13 secs. The higher values of mean accepted gaps compared to median values indicate that few drivers (cautious drivers) accepted larger gaps compared to the other drivers. The mean value of mean accepted gaps showed slightly larger difference; 1.29 secs for uncongested data versus 1.40 secs for congested data, respectively. Differences were not observed in the median values of accepted time gaps, 1.13 secs for the two field data sets.

Additionally, the mean values of trailing and leading gaps were evaluated. The mean value of trailing gaps, 1.35 secs was found to be higher than the mean value of leading gaps, 1.24 secs in both the datasets. Drivers were found to be more cautious in accepting the trailing gaps as compared to the leading gaps. This is due to the reason that the trailing gaps are harder to perceive compared to leading gaps. Mean value of leadings gaps, 1.34 secs is higher under congested conditions as compared to uncongested conditions, 1.15 secs. Similarly, mean value of trailing gaps, 1.47 secs is higher under congested conditions as compared to uncongested conditions, 1.23 secs. Time gaps for congested conditions are lower compared to uncongested conditions as time gaps are determined using speeds and the average speeds of the vehicles under congested conditions are lower as compared to uncongested conditions.

Table 2 which represents both consistent and inconsistent driver behaviors, it is observed that the mean value of accepted gaps is slightly lower than the mean values of mean and median rejected gaps under both uncongested and congested datasets and for both the leading and trailing gaps. The mean values for accepted gaps are lower than the mean and median rejected gaps in the range of two to 10% under uncongested conditions, whereas under congested condition mean values of accepted gaps are lower than the mean and median rejected gaps in the range of 18 to 22%. The difference in the above ranges between uncongested and congested condition is due to the fact that time gaps are rejected at lower speeds under congested conditions compared to uncongested conditions.

Table 3 represents the behavior of consistent drivers i.e. drivers that reject gaps that were smaller than the accepted gaps. For accepted gaps, it is observed that the mean values are slightly higher than the median values in both the datasets except for one case. The higher values of mean compared to the median indicate that some drivers accept bigger gaps and these may be referred to as conservative drivers. It is also observed that the mean and median values of accepted time gaps are higher under congested conditions as compared to uncongested conditions. The mean values of mean accepted gaps and median accepted gaps under uncongested conditions equal 1.19 secs and 1.16 seconds, respectively, and the mean values of mean accepted gaps and median accepted gaps under congested conditions equal 1.41 secs and 1.31 seconds, respectively. This is because the speeds are lower in congested conditions as compared to uncongested conditions, therefore, the time gaps are higher.

The mean and median values for trailing time gaps are higher as compared to leading time gaps for both uncongested and congested conditions and for both adjacent lane to shoulder lane and shoulder lane to adjacent lane maneuvers. This is due to the fact that the drivers have a better perception of leading gaps as compared to trailing gaps. For trailing gaps, drivers look for the assumed follower either in the rear view mirror or over the shoulder, therefore, the perception of leading gaps is better than the trailing gaps.

Further, from Table 3 it is observed from the mean and median values of accepted gaps under both the congested and uncongested traffic conditions and for both the leading and trailing gaps, drivers accept larger gaps for making a lane change from adjacent to the shoulder lane (towards the right) and exiting the highway as compared to the drivers making a lane change from the shoulder lane to the adjacent lane (towards the left) and entering the main lanes of the highway.

Analysis of Table 3, for consistent driver behavior, shows that the mean values of accepted gaps are greater by seven to 26% greater compared to the mean values of LRLA when mean values are individually compared for each column. Similar values are observed for median values of accepted gaps when compared to the median values of LRLA when compared for each column.

For consistent driver behavior, it is observed that the difference in accepted gaps is similar under uncongested and congested datasets but difference exists between leading and trailing gaps. For leading gaps, for both the traffic conditions, the difference ranges from seven to 10% in leading gaps whereas the difference for trailing gaps ranges from 28 to 34%. This shows that for consistent drivers, there is difference in accepted gaps between leading and trailing gaps and a difference in the values between the accepted and rejected gaps between the leading and trailing gaps. For trailing gaps, drivers tend to reject larger values of gaps as compared to leading gaps. This is consistent with the findings that the accepted gaps are larger for the trailing gaps as compared to the leading gaps.

Moreover, the mean values of leading and trailing accepted gaps are higher for the adjacent to shoulder lane maneuvers compared to the shoulder lane to the adjacent lane maneuvers in both the datasets. This is due to the fact that the frequency of lane changes is higher from the shoulder lane to the adjacent lane (777) as compared to from the adjacent lane to the shoulder lane (199). Due to the higher frequency, drivers may take higher risks and accept smaller gaps. Table 3 presents the statistics of accepted gaps which also shows trends similar to Table 2.

From analyzing Tables 2 and 3 it was concluded from the mean values of the accepted gaps and mean values of mean, median and LRLA rejected gaps that around 30% of drivers showed inconsistent behavior. To represent the critical gap values for consistent driver behavior in the datasets, values in Table 3 were used to estimate the critical gap values and their distributions which are discussed in the section on ‘Critical Gaps’.

Figure 2 shows the frequency distribution of accepted gaps, subset of accepted gaps (using LRLA criteria) indicated by accepted gaps for LRLA, and rejected gaps (mean, median and LRLA) for leading (Figure 2a) and trailing (Figure 2b) time gaps between the adjacent lane and the shoulder lane for uncongested traffic conditions. Figure 3 and Figure 4 presents the cumulative percentage of the above distributions. Time gap data is grouped into 0.5 second intervals. Gamma and lognormal distributions were attempted to fit the accepted gap data for LRLA criteria, and KS test was used to check the goodness of fit for the distributions. It is observed that the null hypothesis (H_0) was rejected for lognormal distribution for most of the maneuvers. The gamma distribution fitted well for the accepted gaps (for LRLA) for all the maneuvers. Table 4 presents values of $KS_{Observed}$ for the field data and KS_{Table} for gamma distribution. The field data in Table 4 shows that a gamma distribution with a shape parameter (α) ranging from 1.65 to 2.42 and scale parameter (β) ranging from 0.54 to 0.74 could be used for accepted gaps corresponding to LRLA in microscopic traffic simulation models under uncongested and congested traffic conditions for mandatory lane changes.

Rejected Gaps

Table 2 presents the statistics of mean and median rejected gaps. It is observed that the mean value of mean rejected gaps, 1.55 secs and mean value of median rejected gaps, 1.39 secs are greater than the mean value of corresponding mean accepted gaps, 1.34 secs. This can also been observed in Figure 3 (a) and Figure 4 (a) that the cumulative frequency of mean rejected gaps and the median rejected gaps is towards the right of the cumulative frequency distribution of the corresponding accepted gaps. Equation 1, therefore, does not hold for all mandatory lane change maneuvers as 30% of drivers in the NGSIM data violate the inequality when the mean and median values of rejected gaps are evaluated. If the gaps rejected by the drivers are higher than the accepted gaps then the gaps were rejected because of several reasons and these reasons were other than collision avoidance. By general definition as indicated in Equation 1, the rejected gaps should be smaller than the accepted gaps for consistent driver behavior; therefore, the mean and median rejected gaps should not be utilized for the estimation of critical gap distribution. To achieve realistic gap rejection behavior of drivers for estimation of critical gaps, the rejected gaps should be less than the accepted gaps. The largest rejected gap less than the accepted gap (LRLA), therefore, represents the realistic case of gap rejection for consistent drivers.

Table 3 presents the statistics of mean and median of LRLA gaps. It is observed that the mean value of mean rejected gaps, 1.09 secs and mean value of median rejected gaps, 1.05 secs are less than the mean value of corresponding mean accepted gaps, 1.30 secs. This is contrary to the observation in Table 2 and explained in the above paragraph. Table 3, therefore, presents the results for consistent drivers which hold the inequalities in Equations 1 and 2.

For rejected gaps, it is observed that the mean values of LRLA for trailing gaps are shorter than the leading gaps for all pairs of leading and trailing gaps. This is due to the reason that leading gaps are easier to perceive compared to trailing gaps, therefore, a higher number of shorter trailing gaps are rejected. From Table 3 it is observed that the difference between the accepted and LRLA is higher for trailing gaps as compared to the leading gaps. It is also observed that the difference between the accepted and LRLA is larger for congested conditions as compared to the uncongested conditions. This shows that drivers are extra cautious as the vehicles are close to each other under congested conditions.

It is also observed, for trailing gaps, that the median values of LRLA are larger than the mean values except one case, whereas for leading gaps the median values of LRLA are shorter than the mean values. This indicates that for trailing gaps few drivers reject very smaller values of gaps as compared to the driver population and for leading gaps few drivers reject very large gaps as compared to the driver population.

Similar to the results in Table 2, it is observed from Table 3 that the drivers reject larger gaps when making a lane change from the adjacent lane to the shoulder lane (towards right) as compared to when making a lane change from the shoulder lane to the adjacent lane (towards left). This shows the gap rejection behavior of drivers and that they have a better perception of gaps when making a lane change towards the left as compared to making a lane change towards the right.

From Figure 3 (b) and Figure 4 (b), it can be seen that the cumulative frequency distribution of LRLA is towards the left of the corresponding accepted gaps. These figures also clearly indicate the difference in accepted gaps for trailing and leadings gaps; that trailing gaps are larger than the leading gaps.

Critical Gaps

Critical gaps were estimated using LRLA and the corresponding accepted gaps presented in Table 3 that are consistent with Equation 1. Table 4 presents the values of critical time gaps determined using MLE. The results indicate that the values of trailing critical gaps are higher than the values of leading critical gaps in both the uncongested and congested traffic conditions. This result, similar to the finding for accepted gaps which shows that the drivers making a lane change prefer to have larger trailing gaps than the leading gaps. Similarly, critical time gaps are determined to be greater under congested conditions as compared to uncongested conditions. This finding may also be because under congested conditions, presented in Figure 1 (b), data was available until 355 feet before the off-ramp. As the drivers approach the off-ramp when traveling on the shoulder lane, the accepted gaps get shorter as the drivers approached the lane drop.

For critical gaps, determined using MLE, it is observed that the trailing and leading critical gaps are 22% and 15% greater, respectively, during congested conditions compared to the uncongested conditions. It is also observed that under uncongested and congested conditions, the average value of critical gaps from the adjacent lane to the shoulder lane is about 16 to 18% higher than the average value from the shoulder lane to the adjacent lane. This shows that drivers merging from the on-ramp and merging to the adjacent lane accept shorter gaps and do not reject larger gaps in comparison to the drivers exiting from the off-ramp.

Critical gaps, based on the distribution of accepted and rejected gaps were assumed to be gamma distributed as critical gaps were determined using values of accepted gaps. Table 6 presents the parameters of critical gap distribution estimated using MLE. These parameter values can be used to generate gamma distributed critical gaps which can be assigned to drivers in microscopic simulation models.

In a previous study (12) for estimating critical gaps using the largest rejected gaps, it was found that the largest rejected gaps are greater than the accepted gaps, and as a result, critical gaps estimated using the largest rejected gaps lie between the mean value of the accepted gaps and the mean value of the largest rejected gaps. The critical gaps estimated using the largest rejected gaps and the accepted gaps can be used to determine the threshold of the largest rejected gaps. The threshold of the largest reject gap represents the value above which the drivers will not accept a gap. To understand the utilization of the critical gaps estimated using LRLA and the critical gaps estimated using the largest rejected gaps (LR) (12), the mean values of different gaps is presented below:

LRLA = Mean value of largest rejected gaps less than the accepted gaps = 1.09 seconds (determined using results from Table 3),

C_{LRLA} = Critical gap estimated using LRLA, ($LRLA < C_{LRLA} < A$) = 1.19 seconds (from Table 5),

A = Mean value of accepted gaps = 1.30 seconds (determined using results from Table 3),

C_{LR} = Critical gap estimated using the largest rejected gaps (LR > C_{LR} > A) = 1.72 seconds (12), and

LR = Mean of largest rejected gaps = 4.00 seconds (from data set).

The mean value of largest rejected gaps, 4.0 secs is much larger than the mean value of accepted gaps, 1.3 secs. This represents gap rejection due to reasons other than collision avoidance such as the location was not the intended position of lane change, etc.. The largest rejected gap less than the accepted gap represents rejected gaps because of unsafe lane change conditions such that cause collision if lane change is performed. The mean value of accepted gaps is greater than mean value of LRLA and provides a safe lane change when lane change is made. Critical gaps based on the largest rejected and accepted gaps represent the threshold of gap rejection which shows that drivers reject gaps which are larger than the accepted gap due to reasons other than safe lane change maneuvers. If the available gap is greater than C_{LRLA}, then it is acceptable for a safe lane change maneuver.

C_{LR} can be used in gap rejection so that an available gap that is larger than C_{LR}, the lane change can be delayed and the driver does not make a lane change as soon as it finds the first available gap. This is important because if only C_{LRLA} or collision avoidance conditions are used in gap acceptance models for mandatory lane changes then the vehicles will make lane changes the moment these conditions are met. This is not a realistic representation of lane changes in terms of intended location of lane change. By using $C_{LR,}$ a threshold of largest rejected gap can be induced in gap acceptance models. This will add stochasticity in driver lane change behavior by using such gap acceptance models. Using C_{LR} along with C_{LRLA} in gap acceptance algorithms will provide realistic representation of both gap acceptance and gap rejection behavior of the drivers.

Maximum Likelihood Estimation

Table 6 presents the estimation of mean values of critical gaps using the MLE method. The MLE method uses an iterative process to maximize the partial derivatives of the likelihood of the critical gap with respect to the parameters of gamma distribution; shape (α) and scale (β). Earlier studies assumed exponential (6), lognormal (7), and normal (3) distributions for the critical gaps, however, from the data analyzed for this report and in a paper (12), critical gaps are found to be gamma distributed based on accepted and rejected gaps. Maximum likelihood estimated the parameters of the gamma distribution i.e. the mean and the variance of the distribution. MLE uses a rigorous process to determine the critical gaps, therefore, this method is recommended for estimating critical gaps.

Table 6 presents the mean values of critical gaps. Mean leading and trailing critical gaps for the combined datasets (uncongested and congested) and for the both the lane change maneuvers combined, were found to be 1.17 and 1.26 seconds, and the variance to be 0.82 and 0.87 seconds^2, respectively. These values of distribution parameters may be used for generating gamma distributed critical gaps in different traffic conditions and for different maneuvers in microscopic simulation models. Figure 5 shows the CDF of gamma distributed critical gaps determined using MLE, accepted gaps and LRLA for both leading and trailing gaps for the adjacent lane to the shoulder lane maneuver.

CONCLUSIONS AND RECOMMENDATIONS

This research proposes critical gaps and its distribution parameters which can be used to replicate drive gap acceptance behavior in traffic simulation models. Mandatory lane change maneuvers observed in the field data do not always follow collision avoidance conditions and gaps are rejected because of reasons that than collision avoidance. Critical gaps, therefore, are proposed as a driver behavior characteristic that represents realistic driver behavior.

Many researchers have presented estimation methods for critical gaps that are useful for vehicles entering from a minor street to a major street on unsignalised intersections and for over-passing vehicles on two lane roads. This report presents gap acceptance and estimation of critical gaps for mandatory lane changes on a multilane freeway with an on-ramp, an off-ramp and a lane drop. Critical gaps, in this report, are estimated using both the accepted and the rejected gaps.

Contrary to unsignalized intersections, where a driver on a minor street comes to a complete stop and then looks for an appropriate gap to merge on the main street, on freeways drivers are in motion while looking for an appropriate gap to make a safe lane change. From the field data, it is not possible to determine the exact reason for rejecting a gap. A driver can reject a gap because of several reasons such as to avoid collision, not reached the intended location of a lane change, misperception of a gap, significant difference in relative speed, etc. Therefore, rejected gaps are categorized into: mean rejected gaps, median rejected gaps and the largest rejected gaps less than the accepted gaps (LRLA). A critical gap lies between the rejected gap and the accepted gap. Average mean value of mean rejected gaps and median rejected gaps are larger than the average mean values of corresponding accepted gaps, for leading and trailing gaps, for the subject vehicle in interaction with both the assumed leader and the assumed follower. Therefore, mean rejected gaps and median rejected gaps are not recommended for the estimation of critical gaps for freeway conditions. In case of largest rejected gap less than the accepted gap, all the rejected gaps are less than the accepted gaps so estimated critical gap lies between the rejected gaps and the accepted gaps. LRLA are, therefore, recommended for the estimation of critical gaps.

Gamma distribution is proposed for accepted time gaps as it fits well for both leading and trailing accepted time gaps for both uncongested and congested traffic flow conditions. It can be concluded that gamma distribution represents the gap acceptance behavior of drivers during a lane change and can be used in microscopic traffic simulation models. Lognormal distribution is not recommended for accepted gaps as the null hypothesis is rejected for most of the maneuvers, and therefore, the distribution fails to represent realistic driver behavior in gap acceptance. Similarly, rejected gaps follow the gamma distribution.

Mean values of estimated critical time gaps have been determined using a stochastic method. Trailing critical gaps are found to be higher than the leading critical gaps. This result signifies that the drivers are more sensitive to trailing critical gaps compared to leading critical gaps. Results also show that for mandatory lane changes critical gaps are larger for congested conditions compared to uncongested conditions.

Previous studies have assumed a lognormal distribution for the critical gaps, however, this was not found in the data sets analyzed for this report. As the accepted and rejected gaps follow a gamma distribution, therefore gamma distribution is a more logical choice for estimation of critical gap parameters using the MLE method. MLE method estimates distribution parameters of critical gaps and is also recommended for estimation of critical gaps for mandatory freeway lane change maneuvers. Leading and trailing critical gaps for the combined datasets (uncongested and congested) and for the both the maneuvers combined (from adjacent to shoulder lane and shoulder lane to adjacent lane) were determined by MLE.

Results obtained from the distribution of accepted time gaps and estimated critical time gaps can be used in traffic simulation models to generate values of accepted gaps and critical gaps for mandatory lane changes. Critical gaps estimated using the largest rejected gaps that are less than the accepted gaps can be used along with the critical gaps estimated using largest rejected gaps to get realistic representation of driver gap acceptance and gap rejection behavior. This procedure can also be used in existing lane change models that utilize the collision avoidance check for allowing a lane change. Also, leading and trailing accepted and rejected gap values from the field data may be used for calibrating traffic simulation models. Critical gap values may also be used to identify the point where drivers start to make a lane change and traffic signs can be provided on the freeway. Further study is recommended to estimate critical gaps for discretionary lane changes and multiple lane changes. Further research can be carried out on the development of a simulation model based on critical gaps, gap rejection and gap acceptance based on field data and using the concepts of LRLA and LR.

ACKNOWLEDGEMENTS
We acknowledge the NGSIM data sets provided by Cambridge Systematics. The support of the University Transportation Center is also appreciated.

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Table 1. Terminologies used in the report

- Mandatory lane changes: the essential lane changes that drivers make to exit from the off-ramp, enter main lanes from the on-ramp, or merge to the adjacent lane to avoid lane drop.
- Subject vehicle: the vehicle that intents to make a lane change.
- Assumed leader: the immediate leader to the subject vehicle in the target lane.
- Assumed follower: the immediate follower to the subject vehicle in the target lane.
- Leading time gaps: calculated as the distance between the rear bumper of the assumed leader and the front bumper of the subject vehicle over the speed of the subject vehicle.
- Trailing time gaps: calculated as the distance between the rear bumper of the subject vehicle to the front bumper of the assumed follower over the speed of the assumed follower.
- Shoulder lane: the rightmost lane.
- Adjacent lane: the lane next to the shoulder lane (Figure 1).
- Leading accepted gap: the time gap between the subject vehicle and the assumed leader at a time frame when the center of the front bumper of the subject vehicle just crosses the lane marking.
- Trailing accepted gap: the time gap between the subject vehicle and the assumed follower at a time step when the center of the front bumper of the subject vehicle just crosses the lane marking.
- Rejected leading gap: the time gap between the subject vehicle and the assumed leader between a lane change and five seconds prior to a lane change.
- Rejected trailing gap: the time gap between the subject vehicle and the assumed follower between a lane change and five seconds prior to a lane change. In this report, rejected gaps mean the largest rejected gap less than the accepted gap or LRLA, unless otherwise stated.
- Gaps are classified further as follows:
    - Rejected Gaps
        - Mean rejected gaps – Mean of five rejected gaps prior to a lane change.
        - Median rejected gaps – Median of five rejected gaps prior to a lane change.
        - Largest rejected gap less than the accepted gap (LRLA) - Largest value among the five rejected gaps which is less than the accepted gap.
        - Largest rejected gap – Largest value of rejected gap.
    - Critical Gaps

      C_{LRLA} = Critical gap determined using LRLA and accepted gap (A), ($LRLA < C_{LRLA} < A$),

Table 2. Statistics of Accepted and Rejected Gaps

Statistics	Uncongested Data				Congested Data			
	Adjacent to Shoulder Lane		Shoulder to Adjacent Lane		Adjacent to Shoulder Lane		Shoulder to Adjacent Lane	
	Time Gaps (seconds)							
	Leading	Trailing	Leading	Trailing	Leading	Trailing	Leading	Trailing
Median	1.21* 1.14# 1.55^	1.12 1.16 1.36	1.08 1.10 1.72	1.10 1.03 1.78	1.04 1.61 2.39	1.09 1.68 2.47	1.16 1.64 2.58	1.21 1.72 2.67
Mean	1.31* 1.34#	1.32 1.41	1.23 1.30	1.30 1.38	1.36 1.69	1.41 1.72	1.35 1.72	1.46 1.87
Std Dev.	0.88~ 0.83@	0.93 0.97	0.72 0.88	0.76 0.73	0.97 0.98	0.93 0.86	0.87 0.94	0.91 0.94
No. of Observations	109		456		90		321	

Note: * are accepted gaps, # are mean rejected gaps, and ^ are median rejected gaps.
~ accepted gaps @ rejected gaps

Table 3. Statistics of Accepted and Largest Rejected Gaps less than the Accepted Gaps (LRLA)

Statistics	Uncongested Data				Congested Data			
	Adjacent to Shoulder Lane		Shoulder to Adjacent Lane		Adjacent to Shoulder Lane		Shoulder to Adjacent Lane	
	Time Gaps (seconds)							
	Leading	Trailing	Leading	Trailing	Leading	Trailing	Leading	Trailing
Median	1.20 **(1.08)**	1.27 **(1.01)**	1.08 **(0.78)**	1.10 **(0.96)**	1.34 **(1.24)**	1.42 **(1.28)**	1.19 **(0.97)**	1.28 **(1.09)**
Mean	1.28[1] **(1.19)[2]**	1.33 **(1.04)**	1.03 **(0.94)**	1.13 **(0.84)**	1.43 **(1.31)**	1.58 **(1.23)**	1.26 **(1.14)**	1.37 **(1.03)**
Difference (1-2)	**0.09**	**0.29**	**0.09**	**0.19**	**0.12**	**0.35**	**0.12**	**0.34**
Std Dev.	0.89 **(0.97)**	0.72 **(0.88)**	0.80 **(0.78)**	0.85 **(0.82)**	0.97 **(0.72)**	0.86 **(093)**	0.87 **(0.92)**	0.78 **(0.94)**
No. of Observations	63		264		62		297	

Note: Values in parenthesis are the largest rejected gaps less than the accepted gaps (LRLA)

Table 4. KS Test for Accepted Time Gaps (for LRLA)

Maneuver		Parameters		KS Stat.	Leading		Trailing	
		Leading	Trailing		KS Obs.	H_0	KS Obs.	H_0
Uncongested Data	Adjacent to Shoulder Lane	α = 2.07 β = 0.62	α = 2.42 β = 0.54	0.1680	0.1011	**NR**	0.1050	**NR**
	Shoulder to Adjacent Lane	α = 1.65 β = 0.63	α = 1.75 β = 0.65	0.0836	0.0326	**NR**	0.0618	**NR**
Congested Data	Adjacent to Shoulder Lane	α = 1.98 β = 0. 72	α = 2.12 β = 0.74	0.1287	0.0921	**NR**	0.0962	**NR**
	Shoulder to Adjacent Lane	α = 1.71 β = 0.73	α = 1.91 β = 0.72	0.0975	0.0478	**NR**	0.0584	**NR**

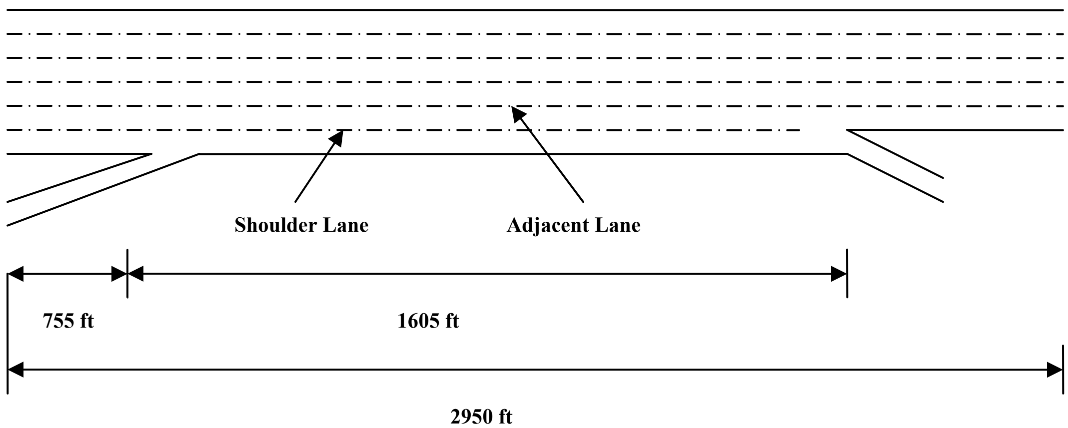
Note: NR = Not Rejected

Table 5. Critical Time Gaps estimated using Accepted Gaps and Largest Rejected Gaps Less than the Accepted Gaps

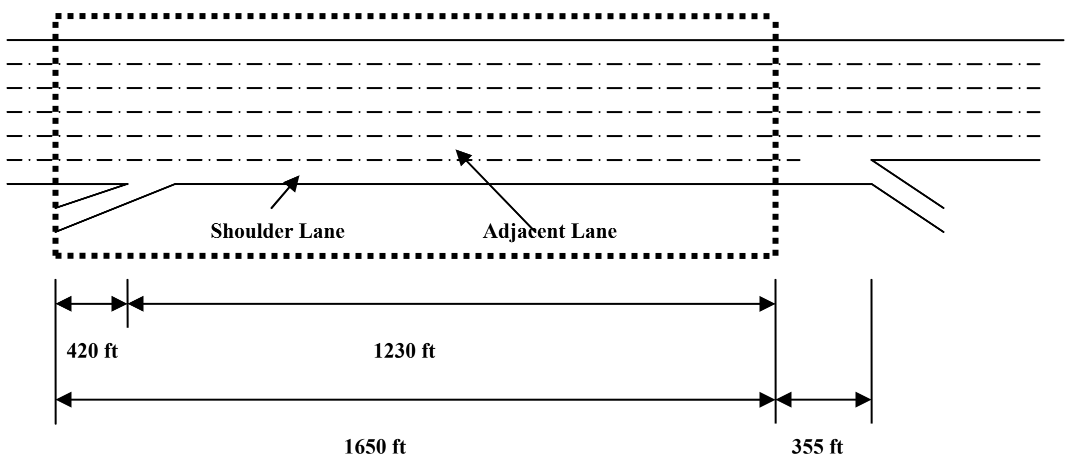
Lane Change Maneuver From – To	Uncongested Data		Congested Data		Average
	Leading	Trailing	Leading	Trailing	
Adjacent to Shoulder Lane	1.22	1.14	1.35	1.41	**1.28**
Shoulder to Adjacent Lane	0.98	1.02	1.19	1.24	**1.10**
Average	**1.10**	**1.08**	**1.27**	**1.32**	**1.19**

Table 6. Gamma Distribution Parameters for Critical Gaps

Maneuver	Uncongested Data						Congested Data					
	Leading			Trailing			Leading			Trailing		
	α	β	Var ($\alpha \beta^2$)	α	β	Var ($\alpha \beta^2$)	α	β	Var ($\alpha \beta^2$)	α	β	Var ($\alpha \beta^2$)
Adjacent to Shoulder Lane	1.54	0.80	**0.98**	2.42	0.47	**0.53**	1.88	0.71	**0.94**	1.98	0.71	**0.98**
Shoulder to Adjacent Lane	1.49	0.61	**0.57**	1.10	0.92	**0.94**	1.78	0.66	**0.77**	1.57	0.79	**0.98**

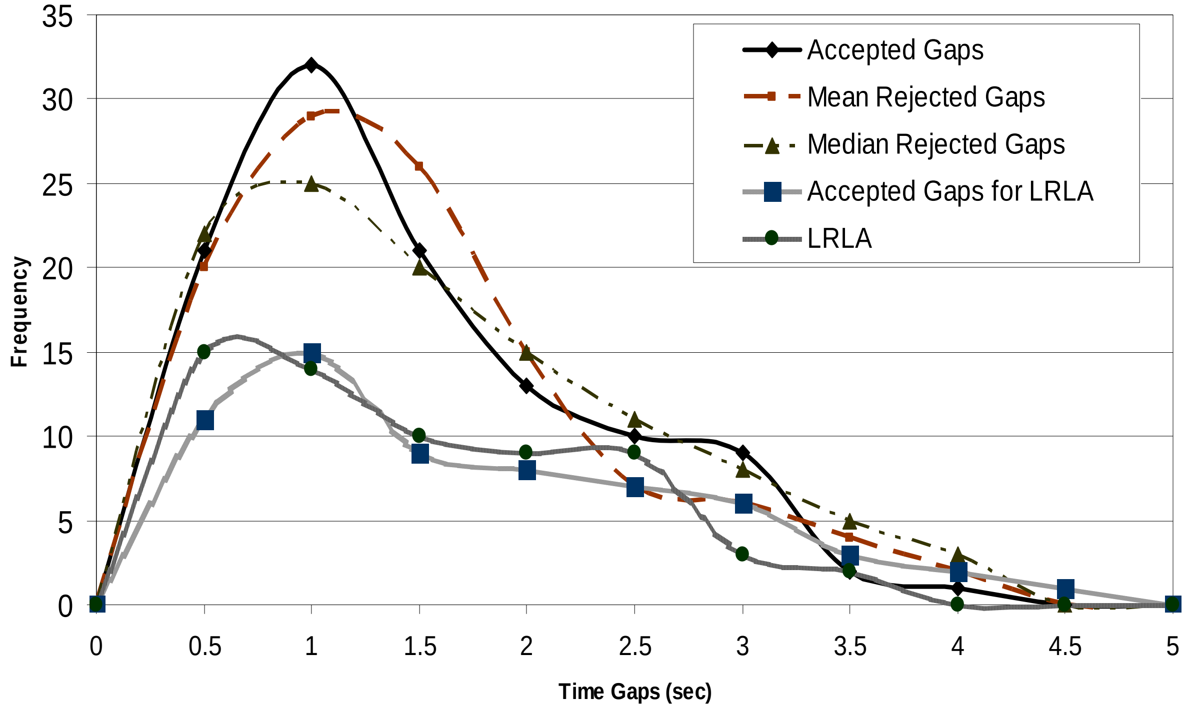


a) Uncongested Data with Six Main Lanes, an On-Ramp and an Off-Ramp.

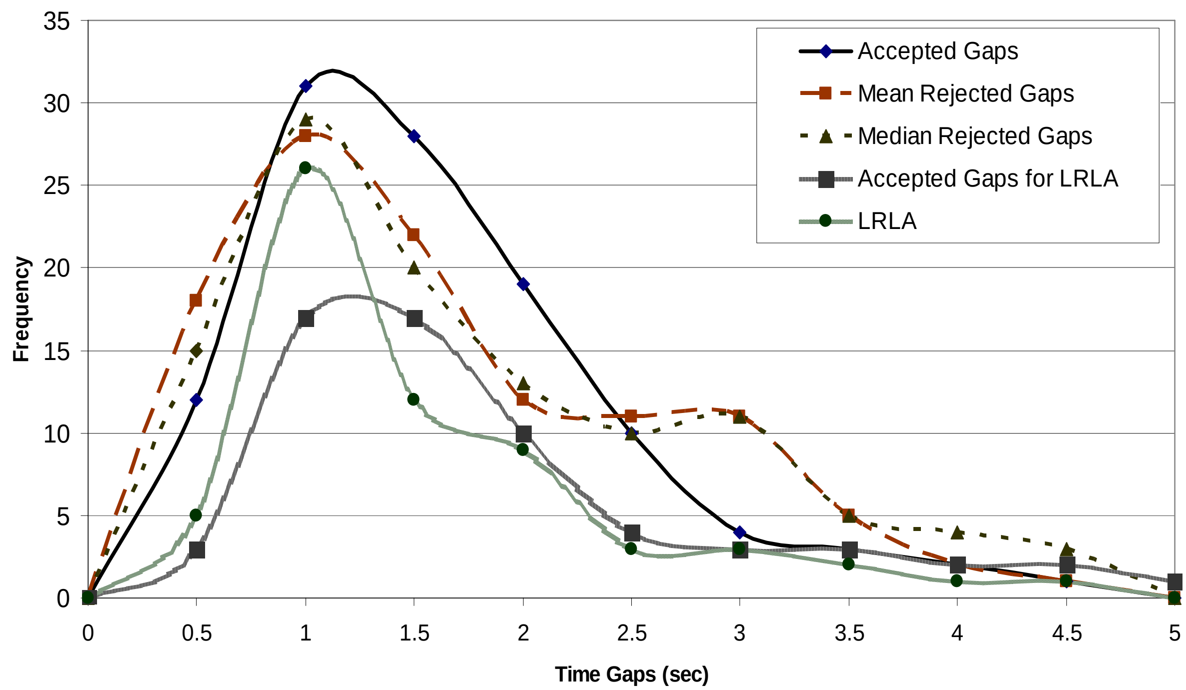


b) Congested Data with Six Main Lanes and an On-Ramp
(Data collected in the dotted area)

Figure 1. Schematic of Datasite at I-80, Emeryville, California
(Note: Figure not to scale)

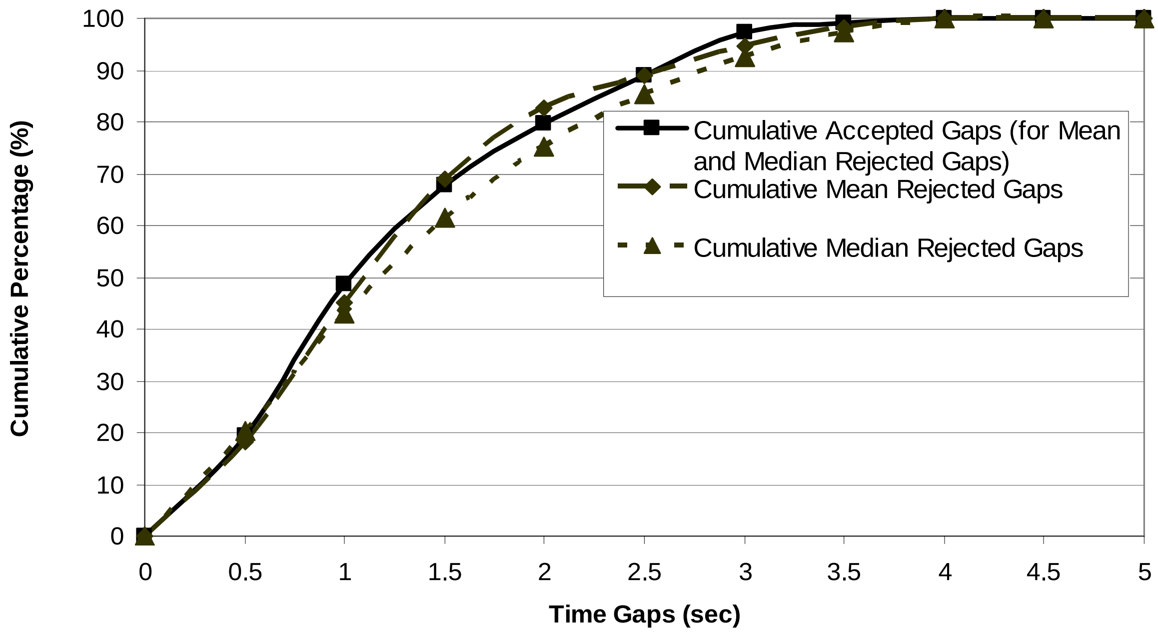


a) Leading Gaps from Adjacent Lane to Shoulder Lane

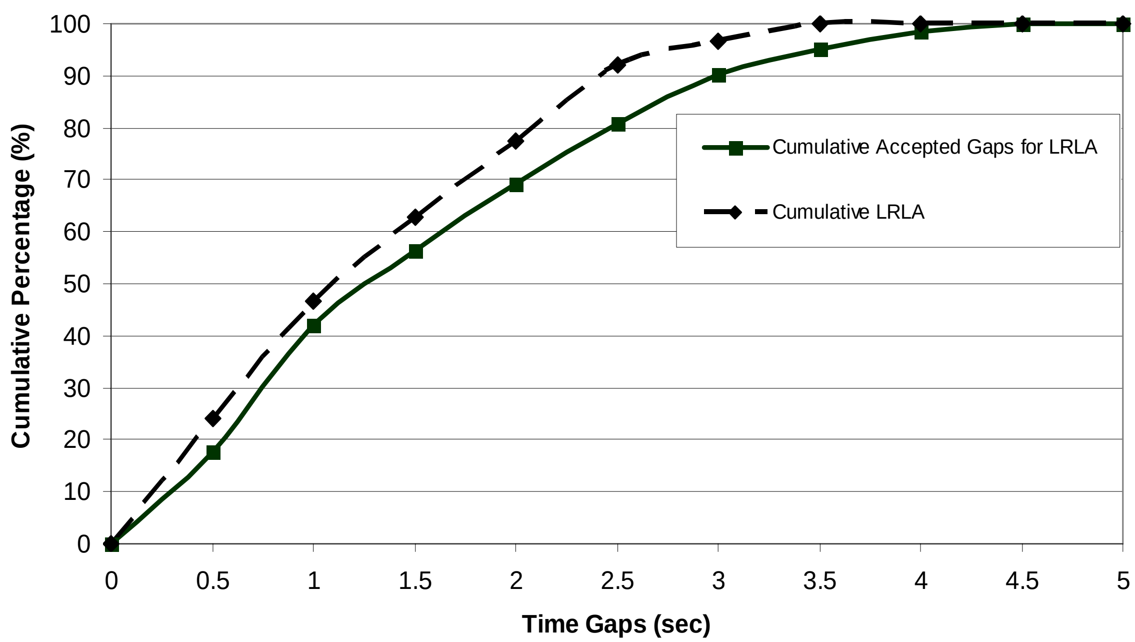


b) Trailing Gaps from Adjacent Lane to Shoulder Lane

Figure 2. Frequency Distribution for Accepted, Mean Rejected, Median Rejected, Accepted Gaps for LRLA, and LRLA

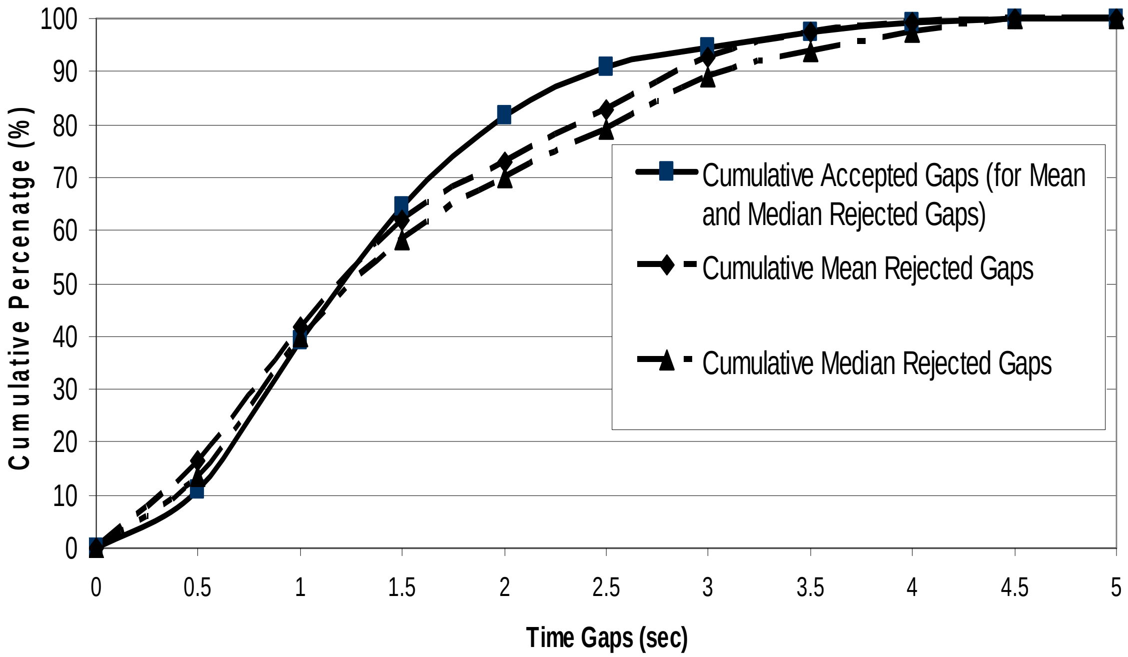


a) CDF of Accepted, Mean Rejected and Median Rejected Time Gaps

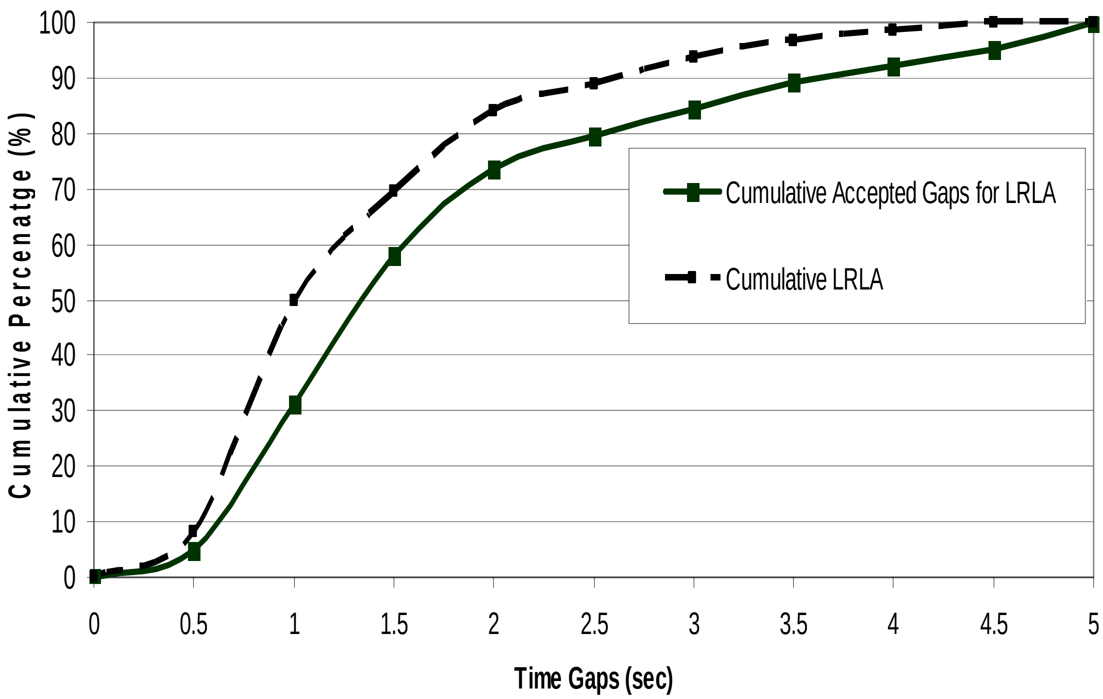


b) CDF of Accepted (for LRLA) and LRLA Time Gaps

Figure 3. CDF of Leading Time Gaps for Uncongested Conditions from Adjacent Lane to Shoulder Lane

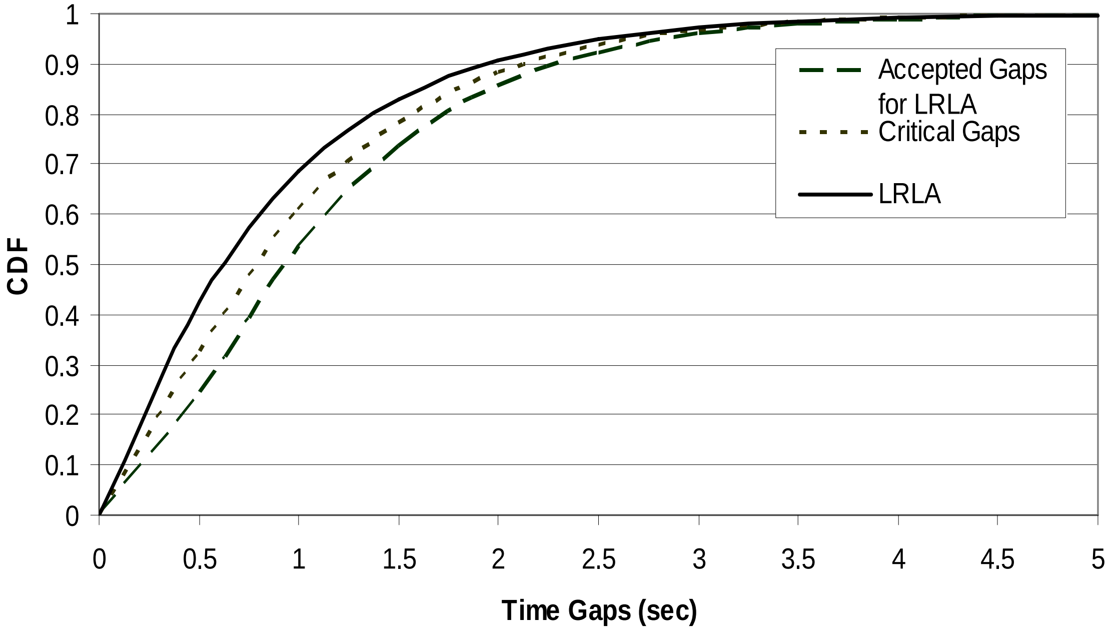


a) CDF of Accepted, Mean Rejected and Median Rejected Time Gaps

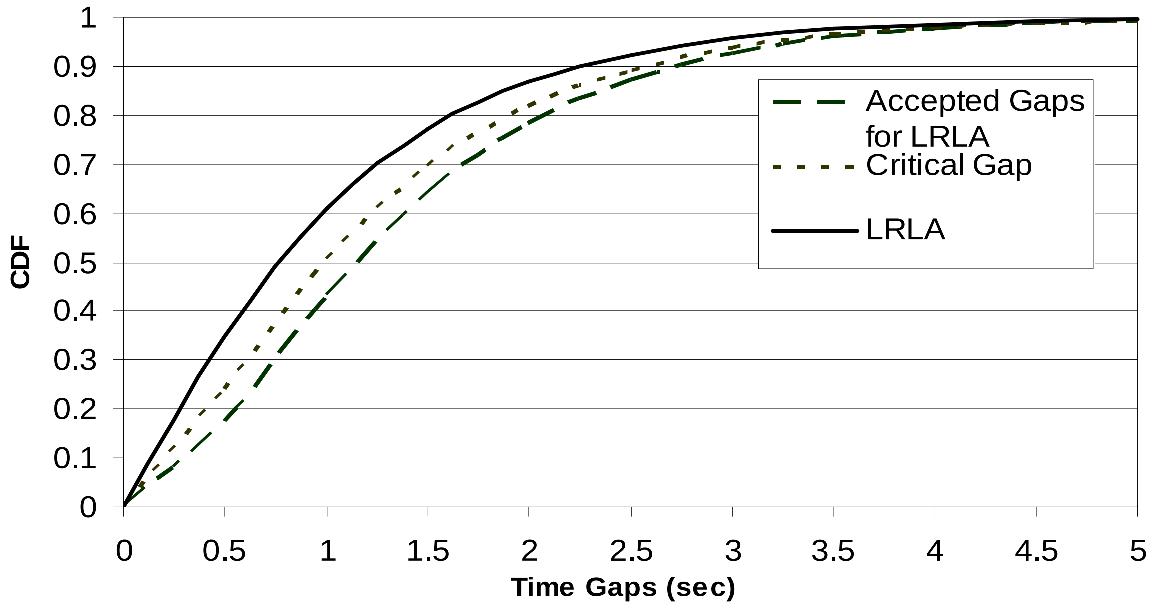


b) CDF of Accepted (for LRLA) and LRLA Time Gaps

Figure 4. CDF of Trailing Time Gaps for Uncongested Conditions from Adjacent Lane to Shoulder Lane



a) Leading Gaps from Adjacent Lane to Shoulder Lane



b) Trailing Gaps from Adjacent Lane to Shoulder Lane

Figure 5. CDF of Gamma Distributed Critical Gaps from Maximum Likelihood Estimation