DESIGN OF CONTINUOUS FRP-STRENGTHENED CONCRETE STRUCTURES ALLOWING MOMENT REDISTRIBUTION

Tim Ibell, University of Bath, UK
Pedro Silva, University of Missouri-Rolla, USA

ABSTRACT

The use of fiber-reinforced polymer (FRP) composites is now a widely-accepted solution for the strengthening of reinforced concrete structures. FRP strengthening schemes offer many well-documented benefits for the retrofit of many existing concrete buildings and bridges. However, the main drawback in using FRP for such purposes is the reduction in ductility that the strengthened structure displays after strengthening. This loss in ductility has led various design guidelines around the world to prohibit any redistribution of bending moments in continuous FRP-strengthened concrete structures. This means that continuous structures, which might have been designed, originally under assumptions of moment redistribution, should be designed for FRP strengthening according to elastic distribution of bending moment. This could lead to onerous conditions for such strengthening schemes, particularly in hogging regions. This paper sets out a rationale for the possible appropriate use of redistribution principles for FRP-strengthened concrete.

KEY WORDS: Moment Redistribution, Fiber Reinforced Polymers (FRP), Continuous Beams

1. INTRODUCTION

In today’s world, protection of the environment is becoming a major consideration in any building project. Sustainability is a key issue in all its various forms, from embodied energy of materials to energy associated with transport, construction and operation. Strengthening of existing structures avoids the need to demolish and replace, enabling the design life of the current structure to be increased.

Extensive application examples and research exist on flexural strengthening of reinforced concrete structures using FRP. The principal advantages in using FRP for such applications lead to a rapid, cost-effective strengthening scheme, so that FRP is now the preferred choice worldwide for retrofit of concrete structures.

However, FRP essentially displays elastic behaviour up to brittle failure. Further, the principal failure mode associated with FRP-strengthened concrete structures is peeling of the FRP from
the concrete, usually in a brittle manner⁶. Therefore, at first glance, it appears unlikely that moment redistribution should be permitted in FRP-strengthened continuous concrete structures.

However, strengthening of existing continuous steel-reinforced concrete structures, which were designed according to moment redistribution principles, could lead to onerous design conditions, particularly in hogging regions. The degree of FRP strengthening which could be required under such circumstances might become excessive. This is because the original elastic hogging moment plus the additional elastic moment (required for strengthening purposes) need now to be resisted at this location, compared with just the original redistributed hogging moment. If it could be shown that sufficient ductility exists in an FRP-strengthened region, it would be beneficial to be able to carry out limited moment redistribution to alleviate this problem.

Further, fundamentally there are two redistribution issues that need to be addressed. The first relates to redistribution of bending moment into an FRP-strengthened region, while the second relates to redistribution of bending moment out of an FRP-strengthened region. This paper addresses both issues and suggests levels of redistribution which could be adopted that directly conform with redistribution principles in existing design codes-of-practice around the world⁷,⁸.

The issue of allowing moment redistribution out of FRP-strengthened zones is discussed in this section. Research in this area is extremely limited, although some work has indeed been conducted⁹. Equally important work has been conducted into the question of ductility¹⁰,¹¹ and rotation capacity¹²,¹³ in FRP-strengthened concrete structures.

If a structure displays ductility, it will also display rotation capacity. However, if a structure displays rotation capacity, it will not automatically display ductility. Therefore, it is the premise of this paper that, in order to define the context within which redistribution should be allowed in FRP-strengthened concrete structures, ductility (rather than mere rotation capacity) should be the over-riding concern. In this way, if ductility can be demonstrated, it is certain that a safe design based on redistribution will result.

### 2. MOMENT-CURVATURE RELATIONS

The beam section shown in Figure 1 is used here to develop example moment-curvature plots with and without FRP strengthening. If one assumes that such a typical concrete structure often carries roughly equal dead and live loading under service conditions, it may be seen that, on average, the service moment acting on such a structure during strengthening is:

$$M_{service} = M_{u \text{ unstrengthened}} / (1.4 + 1.6) \approx 0.30 M_{u \text{ unstrengthened}}$$  \hspace{1cm} \text{Eq. 1}

This ignores any live loading that could be acting. Figure 2 shows theoretical moment-curvature plots for this cross section under increasing quantities of FRP strengthening, which have been added while the section is strained under deadweight. It is clear that as more FRP is added to the section, the post-yield branch stiffens up considerably, and the failure mode changes from FRP delamination to concrete compression failure. In addition, as expected, increasing the FRP area decreases the ductility, \( \mu \) (curvature at the ultimate condition, \( \phi_u \), divided by curvature at steel yielding, \( \phi_y \)), as shown in Figure 3.
Figure 1 Beam cross section

Figure 2 Moment-curvature plots

Figure 3 Ductility ratio under increasing (very high) quantities of FRP

3. DUCTILITY EXHIBITED BY FRP-STRENGTHENED CONCRETE

Based on observations derived from Figure 2, Figure 4 shows a schematic representation of a typical moment–curvature plot for an FRP-strengthened concrete section (dashed line). Also shown is the moment–curvature plot for the un-strengthened section (continuous line) under the service moment given by Eq. 1. The plots are offset horizontally by \( \phi_{\text{datum}} \), corresponding to the datum curvature across the section when the FRP material was added under unpropped conditions. Having removed the datum curvature, the effective curvature across the section may be defined as the additional curvature under strengthened conditions. It may be seen that if the initial moment across the section (at unpropped strengthening) is higher than 30% of the un-
strengthened section capacity, the moment–curvature plots shift horizontally to the left. The effective ductility $\mu$ of the strengthened section may be defined as:

$$\mu = \frac{\phi_u - \phi_{\text{datum}}}{\phi_y - \phi_{\text{datum}}} \quad \text{Eq. 2}$$

It can be shown that this ratio increases as the initial service moment on the concrete structure at strengthening increases. It is undesirable for the initial moment to be too high at strengthening, as this could lead to premature yielding of the steel bars under the higher service-load conditions when fully strengthened. However, it is also clear from the above that it is undesirable for the initial moment to be too low at strengthening, as this could lead to a low level of ductility.

Using this approach, it is then possible to calculate a rational level of ductility that the
strengthened section would display. This new measure of ductility will be evaluated for the design of continuous beams using moment redistribution.

4. MOMENT REDISTRIBUTION LIMITS

How does the level of ductility found in Eq. 3 relate to equivalent levels which are required in present codes-of-practice for conventional reinforced concrete moment redistribution? ACI-318\textsuperscript{7} states that the level of redistribution which may be assumed in a steel-reinforced continuous concrete structure is 1000\(\varepsilon_t\) percent, up to a maximum of 20\%, where \(\varepsilon_t\) is the level of strain in the tension reinforcement, which must be at least 0.0075.

4.1 Sectional Analysis Approach

An analysis approach to justify the amount of redistribution that could be allowed in the retrofit design of continuous FRP-strengthened concrete structures is the sectional analysis approach. It is discussed at large extent in this section. Assuming a singly-reinforced concrete section of effective depth \(d\), such that the strain in the tension steel reaches 0.0075 just as the maximum strain in the concrete reaches 0.003, the curvature at the ultimate limit state is clearly:

\[
\phi_u = \frac{0.0105}{d}
\]

Further, the area of reinforcement, \(A_s\), which is necessary for this strain limit state to develop is given by:

\[
A_s = 0.85 \frac{f'_c}{f_y} \frac{0.003}{0.0075} \beta b d
\]

Where \(b\) is the breadth of the section, \(f'_c\) is the cylinder compressive strength and \(f_y\) is the yield strength of the steel reinforcement. Then, assuming this area of steel, it may be shown that the depth to the neutral axis, \(c\), when the steel first yields is \(c = 0.46d\). The curvature at first yield is then:

\[
\phi_y = \frac{0.0023}{d - 0.46d}
\]

This assumes that the yield strain of the steel reinforcement is 0.0023. Therefore, dividing Eq. 4 by Eq. 6, the curvature ductility ratio is \(\mu = 2.47\). This implies that as long as the ductility ratio across the section is at least 2.47 and the strain in the tension steel is at least 0.0075, moment redistribution should be allowed in conventional steel-reinforced concrete structures.

It is suggested here that similar criteria be imposed on FRP-strengthened sections for moment redistribution out of such locations. It is suggested that as long as the ductility ratio in such sections is at least 2.50, say, and as long as the total strain in the tension steel is at least 0.0075, moment redistribution should be permitted out of such sections.

At strengthening, it may be estimated that the strain in the tension steel bars will be in the region of 30\% of the yield strain. This translates to a strain of roughly 0.0007. To prevent peeling failure, it is usual for the strain limit in the FRP to be taken as about 0.8\%\textsuperscript{4}. At this level of strain on the soffit, the total strain in the tension steel bars would be:
\[ \varepsilon_y = 0.0007 + \frac{(d - c)}{(h - c)} \cdot 0.0080 \]

Eq. 7

If it is assumed that generally \( h \geq 1.10d \) for most large concrete structures, it turns out that for the strain in the steel bars to reach 0.0075, the depth to the neutral axis, \( c \), is about 0.43 \( d \), which is large. If the depth to the neutral axis is less than this, as is usual, the total strain in the steel bars exceeds 0.0075. Therefore, it is clear that the criterion that the steel strain must be at least 0.0075 appears to be met under normal conditions of FRP strengthening. Further, it is clear from Figure 3 that the ductility ratio in the particular FRP-strengthened section (Figure 1) is above this threshold level of 2.50 under practical conditions (FRP percentage less than 0.4%). But, what level of ductility could be hoped to achieve in general from an FRP-strengthened structure and would it exceed 2.50, the required minimum?

If it is assumed that the section is strengthened while a deadweight moment of 30% of the ultimate un-strengthened capacity is applied, it is clear that the value of \( \phi_{\text{datum}} \), defined in Figures 4 and 5, is equal to 30% of the curvature in the un-strengthened section at steel yielding. Under fully-strengthened conditions, it may be shown that the curvature at steel yield, \( \phi_y \), for a strengthened section is given by:

\[
\phi_y = \frac{\varepsilon_y}{d - \left( \sqrt{\left( m_s A'_s + m_s A_s + m_f A_f \right)^2 + 2b \left( d' m_s A'_s + dm_s A_s + hm_f A_f \right) - \left( m_s A'_s + m_s A_s + m_f A_f \right)} \right) b} 
\]

Eq. 8

Where \( \varepsilon_y \) is the steel yield strain, \( d' \) is the depth to the top steel reinforcing layer, \( h \) is the overall depth of section, \( m \) is the relevant modular ratio and \( A \) is the relevant reinforcement area, with subscript \( s \) referring to steel and \( f \) referring to FRP. Further, the curvature at the ultimate condition is given by one of the following two equations, depending on whether concrete crushing or FRP delamination is the critical failure mode:

\[
\phi_u = \frac{\varepsilon_{fu}b}{hb - \left( \sqrt{m_s A'_s + m_s A_s + m_f A_f} \varepsilon_{fu} + m_f A_f \right)^2 + 2b \left( d' m_s A'_s + dm_s A_s + hm_f A_f \right) + m_s A'_s + m_s A_s \varepsilon_{fu} + m_f A_f} 
\]

Eq. 9

or:

\[
\phi_u = \frac{1.8 f_u' \varepsilon_{cu}}{\sqrt{\left( \varepsilon_{cu} E_c m_s A'_s + m_f A_f \right) + 3.6 f_u' \left( d' \varepsilon_{cu} E_c m_s A'_s + \varepsilon_{cu} E_c m_s A_s + h \varepsilon_{cu} E_c m_f A_f \right) - \left( \varepsilon_{cu} E_c m_s A'_s + m_f A_f \right)}} 
\]

Eq. 10

Where \( \varepsilon_{fu} \) is the ultimate design strain for the FRP, \( \varepsilon_{cu} \) is the ultimate design strain for the concrete and \( E_c \) is Young’s Modulus for the concrete. The above values from Eqs. 8, 9 and 10 are then plugged into Eq. 3 to yield the ductility ratio, \( \mu \). Note that it is assumed in the above equations that under strengthened conditions, the tension steel always yields, the compression steel is always elastic and the concrete is elastic if the FRP peels. It may easily be shown that, generally, it is FRP peeling that is the more critical failure mechanism for reasonable quantities
of FRP. Therefore, Eq. 9 is usually more critical than Eq. 10 in terms of ultimate curvature prior to failure.

Using this approach, and plugging in various values across a spectrum of practical scenarios, it turns out that under usual conditions the value of ductility ratio is always above 2.50. This implies that moment redistribution should indeed be allowed to be conducted out of such strengthened zones, based on a minimum level of ductility, and in accordance with the principles laid out in ACI-318, and based on the work of Mattock14.

4.2 Elasto-Plastic Analysis Approach Of equal interest to the sectional analysis approach the plastic analysis approach should also be investigated in order to fully quantify the amount of allowable moment redistribution. In this context the plastic analysis approach relies on investigating the amount of plastic rotation that a FRP-strengthened section must possess in order to properly redistribute the required moments. The elasto-plastic approach is schematically shown in Figure 5 in terms of the solid dashed line.

Assuming a three equal-spans continuous beam and under a uniform applied load, \( w \), the moment at formation of the first hinge may be designated as the ultimate moment of the strengthened section, \( M_u^{\text{strengthened}} \), which is given by:

\[
M_u^{\text{strengthened}} = 0.10\left(1 - \alpha\right) w_u L^2
\]

Eq. 11

Where \( \alpha \) is the amount of redistribution allowed, and \( L \) is the span length. In addition, the amount of plastic rotation that will be required at the supports is given by:

\[
\theta_p = \frac{\alpha w_u L^3}{24 EI}
\]

Eq. 12

As previously described, Eq. 12 was developed based on the elasto-plastic approximation defined in Figure 5. Since the bending stiffness, \( EI \), is just the initial stiffness up to the first-yield curvature, \( \phi_y \), one may rewrite Eq. (4) as:

\[
\theta_p = \left(\frac{\alpha}{1 - \alpha}\right) \frac{10}{24} \phi_y L
\]

Eq. 13

This implies that the required ultimate curvature capacity, \( \phi_U \), of a given section for a given allowable percentage of moment redistribution, \( \alpha \), is:

\[
\phi_U = \phi_y + \left(\frac{\alpha}{1 - \alpha}\right) \frac{10}{24} \frac{\phi_y L}{L_p}
\]

Eq. 14

Where \( L_p \) is the plastic hinge length, which for a three equal-spans continuous beams may be conservatively estimated at 0.10L for a reinforced concrete section. For a FRP-strengthened application a smaller value should be used because the crack width propagation will most likely be smaller and spread over a smaller region. In this paper, the plastic hinge length for a FRP-
strengthened application is conservatively estimated at half, which is 0.05L. As such the curvature ductility capacity of a section must be in the order of:

\[
\mu_\phi = 1 + \left(\frac{\alpha}{1-\alpha}\right) \frac{200}{24}
\]

Eq. 15

For a moment redistribution of up to 7.5% Eq. 15 indicates that the curvature ductility capacity of a given section must be in the order of 1.70. This is significantly smaller than the 2.5 computed using the sectional analysis approach. Conservatively, it is suggested here that should the ductility ratio for a particular situation be found to be equal to or greater than 2.50, a minimum level of moment redistribution should be permitted, namely 7.5%. Should a higher level of redistribution be desired, and if it can be shown that the ductility ratio is greater than 2.50, it is advised that a full moment-redistribution analysis be conducted from first principles to justify the level of moment redistribution.

4.3 Bilinear Analysis Approach This approach is similar to the elasto-plastic approach, but now the moment curvature is based on the bilinear approximation depicted in Figure 5 by the light dashed line. According to this approach the amount of plastic rotation necessary to permit for the allowable moment redistribution initiates at the onset of the yield moment.

As before, assuming a three equal-spans continuous beam and under a uniform applied load, \( w \), the yield moment, \( M_{y \text{ strengthened}} \), at formation of the hinge is given by:

\[
M_{y \text{ strengthened}} = 0.10(1-\alpha) w_u L^2
\]

Eq. 16

The amount of plastic rotation that will be required at the supports is given by:

\[
\theta_p = \frac{\alpha w_u L^3}{24EI} \frac{(3+r/2)}{(8r+3)(1-r)}
\]

Eq. 17

Where \( r \) is the bilinear factor for the post-yield branch. Eq. 17 was developed from first principles for a section with a post-yield stiffness. As previously described, Eq. 12 was developed based on the elasto-plastic approximation defined in Figure 5. Using the same approach to obtain Eq. 13, Eq. 17 may be written as:

\[
\theta_p = \left(\frac{\alpha}{1-\alpha}\right) \frac{10}{24} \phi_y L \frac{(3+r/2)}{(8r+3)(1-r)}
\]

Eq. 18

In this case the required ultimate curvature capacity, \( \phi_u \), is:

\[
\phi_u = \phi_y + \left(\frac{\alpha}{1-\alpha}\right) \frac{10}{24} \phi_y L \frac{(3+r/2)}{(8r+3)(1-r)}
\]

Eq. 19

As before the plastic hinge length for a FRP-strengthened application is conservatively estimated at 0.05L. As such the curvature ductility capacity of a section must be in the order of:

\[
\mu_\phi = 1 + \left(\frac{\alpha}{1-\alpha}\right) \frac{200}{24} \frac{(3+r/2)}{(8r+3)(1-r)}
\]

Eq. 20
This indicates that when the bilinear approach is the elasto-plastic approximation Eq. 20 reverts to the results presented in Eq. 15.

4.4 Discussion of Results Results outlined in Section 4.1 were based on the amount of ductility required to impose a level of steel strain of at least 0.0075 as stipulated by ACI-318 for consideration of moment redistribution. In this case a minimum level of curvature ductility of 2.5 was computed. However, this limit does not consider the amount of plastic rotation necessary to provide for moment redistribution. This was investigated in Sections 4.2 and 4.3, and results are graphically depicted in Figure 6. Referring to Figure 6, it is clear that the sectional analysis approach will govern design. Consequently, it is suggested here that a minimum level of moment redistribution should be permitted, namely 7.5%, provided that the curvature ductility capacity of a section be at least 2.5. If it can be shown that the curvature ductility ratio is greater than 2.50, a higher level of moment redistribution should be considered provided that a full moment-redistribution analysis be conducted as outlined in Section 4.3.

![Figure 6 Curvature Ductility vs. Bilinear Factor](image)

5. CONCLUSIONS

This paper has addressed the question of moment redistribution in FRP-strengthened concrete structures by relating such behavior to the level of ductility at critical sections. In particular, it has been shown that moment redistribution into FRP-strengthened zones should be permitted, and that moment redistribution out of FRP-strengthened zones should be permitted if the ductility ratio across the critical section is at least 2.50.
6. ACKNOWLEDGEMENTS

The authors would like to acknowledge the financial support of the Fulbright Commission for the partial funding of this research.

7. REFERENCES

7. American Concrete Institute, ACI 318. (2002) Building code requirements for structural concrete. ACI, Farmington Hills, USA.