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ENGINEERING POLICY GUIDELINES FOR DESIGN OF DRIVEN PILES

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16. Abstract

These guidelines were developed as part of a comprehensive research program undertaken by the Missouri Department of Transportation (MoDOT) to reduce costs associated with design and construction of bridge foundations while maintaining appropriate levels of safety for the traveling public.
The guidelines were established from a combination of existing MoDOT Engineering Policy Guide (EPG) documents, from the 4th Edition of the AASHTO LRFD Bridge Design Specifications with 2009 Interim Revisions, and from results of the research program. Some provisions of the guidelines represent substantial changes to current practice to reflect advancements made possible from results of the research program. Other provisions were left essentially unchanged, or were revised to reflect incremental changes in practice, because research was not performed to address those provisions. Some provisions reflect rational starting points based on judgment and past experience from which further improvements can be based. All of the provisions should be considered as “living documents” subject to further revision and refinement as additional knowledge and experience is gained with the respective provisions. A number of specific opportunities for improvement are provided in the commentary that accompanies the guidelines.

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prepared for

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by

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Preface

These guidelines were developed as part of a comprehensive research program undertaken by the Missouri Department of Transportation (MoDOT) to reduce costs associated with design and construction of bridge foundations while maintaining appropriate levels of safety for the traveling public. The research program included four broad tasks:

- Task 1 – evaluation of site characterization methods for use in Load and Resistance Factor Design (LRFD) and development of procedures to quantify variability and uncertainty in soil/rock properties,
- Task 2 – evaluation of foundation design methods and completion of a foundation load testing program to improve foundation design,
- Task 3 – evaluation of costs and risks for different LRFD limit states and establishment of appropriate target reliabilities for different classes of roadways/structures, and
- Task 4 – calibration of MoDOT specific resistance factors for design of bridge foundations and development of design guidelines to provide means for implementing the results of the research program.

The research program was conducted by faculty, students, and staff from the University of Missouri and Missouri University of Science and Technology in collaboration with MoDOT personnel and private industry. The research program was completed in Fall 2010. These guidelines, along with several others, serve as the principal deliverables from the research program.

The guidelines were established from a combination of existing MoDOT Engineering Policy Guide (EPG) documents, from the 4th Edition of the AASHTO LRFD Bridge Design Specifications with 2009 Interim Revisions, and from results of the research program. Some provisions of the guidelines represent substantial changes to current practice to reflect advancements made possible from results of the research program. Other provisions were left essentially unchanged, or were revised to reflect incremental changes in practice, because research was not performed to address those provisions. Some provisions reflect rational starting points based on judgment and past experience from which further improvements can be based. All of the provisions should be considered as “living documents” subject to further revision and refinement as additional knowledge and experience is gained with the respective provisions. A number of specific opportunities for improvement are provided in the commentary that accompanies the guidelines.

Disclaimer: The guidelines provided in this document have not been formally adopted by the Missouri Department of Transportation. The opinions, findings, and recommendations expressed in this publication are not necessarily those of the Department of Transportation, Federal Highway Administration. This document does not constitute a standard, specification or regulation.

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751.36 Guidelines for Design of Driven Piles

751.36.1 General

These guidelines address procedures for design of driven piles used as foundations for bridge piers, bridge abutments, roadway signs, and other miscellaneous structures. The guidelines were established following load and resistance factor design (LRFD) concepts. The provisions provides herein are intended to produce foundations that achieve target reliabilities established by MoDOT for structures located on different classes of roadways. The different classes of roadways considered include minor roads, major roads, major bridges costing less than $100 million, and major bridges costing greater than $100 million. Additional background regarding development of these provisions and supportive information regarding use of these provisions is provided in the accompanying commentary [link].

751.36.1.1 Accuracy Required

All capacities shall be taken to the nearest 1 (one) kip, loads shown on plans.

751.36.1.2 Steel Pile

Steel piling shall be ASTM A709 (Grade 36) unless structural analysis or drivability analysis requires ASTM A709 (Grade 50) steel.

751.36.1.3 Test Pile

Length shall be pile length + 10'.
When test piles are specified to be driven-in-place they shall not be included in the number of piles indicated in the "PILE DATA" Table.

751.36.1.4 Load Test Pile

When Load Test Pile are specified, the nominal resistance value shall be determined by an actual load test.
For preboring for piles see Sec 702.

751.36.1.5 Preliminary Geotechnical Report Information

The foundation can be more economically designed with increased geotechnical information about the specific project site. Soil information should be reviewed for rock or refusal elevations. Auger hole information and rock or refusal data are sufficient for piles founded on rock material to indicate length of piling estimated. Standard Penetration Test information is especially desirable at each bent if friction piles are utilized or the depth of rock exceeds approximately 60 feet.

751.36.1.6 Geotechnical Redundancy

A nonredundant pile group is a pile group of less than five piles. Resistance factors should be reduced by 20% for nonredundant pile groups. Greater reductions (additional 20%) should be considered when single pile supports an entire bridge pier.

751.36.2 Steel Pile Characteristics

HP Size

Section	Area
HP 10 x 42	12.35 sq. in.
HP 12 x 53	15.58 sq. in.
HP 14 x 73	21.46 sq. in.

The HP 10 x 42 section should generally be used unless a heavier section produces a more economical design or required by a Drivability Analysis. The same size pile must be used for all footings on the same bent. Pile size may vary from bent to bent.

Shell Cast In Place Pipe Pile (CIP) Size

Diameter	Min. Wall Thickness
14 inch	0.250 inch
16 inch	0.375 inch

The wall thickness shown above is the minimum wall thickness required to meet the structural design requirements. The contractor shall determine the pile wall thickness required to avoid damage during driving or after adjacent piles have been driven but not less than the minimum specified.

Minimum tip elevation must be shown on plans. Criteria for minimum tip elevation shall also be shown. The following information shall be included on the plans:

> "Minimum Tip Elevation is required _______________." Reason must be completed by designer such as:
>
> - for lateral stability
> - for required tension or uplift pile capacity
> - to penetrate anticipated soft geotechnical layers
> - for scour
> - to minimize post-construction settlements
> - for minimum embedment into natural ground

751.36.2.1 Pile Tips

Pile tip reinforcement shall be used if specified on the Design Layout. Use of pile tips should be indicated if directed by the Geotechnical report. The need for pile tips should also be reviewed if 50 ksi is required pile strength for design loadings.

751.36.3 General Design Procedure and Limit States

- Structural Analysis
- Geotechnical Analysis
- Drivability Analysis

751.36.3.1 Design Procedure Outline

- Determine foundation load effects from the superstructure and substructure for Service, Strength and Extreme Event Limit States.
- If applicable, determine scour depths, liquefaction information and pile design unbraced length information.
- Determine if downdrag loadings should be considered.
- Select preliminary pile size and pile layout.
- Perform Pile Soil Interaction (DRIVEN) Analysis. Estimate Pile Length and pile capacity.
- Based on pile type and material, determine Resistance Factors for Structural Strength (ϕ_s).
- Determine:
  - Maximum axial load effects at toe of a single pile
  - Maximum combined axial & flexural load effects of a single pile
  - Maximum shear load effect for a single pile
  - Uplift pile reactions

- Determine Nominal and Factored Structural Resistance for single pile
    - o Determine Structural Axial Compression Resistance
    - o Determine Structural Flexural Resistance
    - o Determine Structural Combined Axial & Flexural Resistance
    - o Determine Structural Shear Resistance
- Determine method for pile driving acceptance criteria
- Determine Resistance Factor for Geotechnical Strength (ϕ_G).
- If other than end bearing pile on rock or shale, determine Nominal Axial Geotechnical Resistance for pile.
- Determine Factored Axial Geotechnical Resistance for single pile.
- Determine Nominal pullout resistance if pile uplift reactions exist.
- Check for pile group effects.
- Check Drivability of pile using the Wave equation
- Review Pile Soil Interaction (DRIVEN) Analysis and pile lengths
- Show proper Pile Data on Plan Sheets.

751.36.3.2 Resistance Factor for Structural Strength (ϕ_S)

Pile structural resistance factor for axial resistance in compression and subject to damage due to severe driving conditions where use of a pile tip is necessary:

Metal Shells - 0.60
H-Piles - 0.50

Pile structural resistance factor for axial resistance in compression under good driving conditions where use of pile tip is not necessary:

Metal Shells - 0.70
H-Piles - 0.60

Pile structural resistance factor for combined axial and flexural resistance of undamaged piles:

Axial resistance factor for H-Piles - 0.70
Axial resistance for Metal Shells - 0.80
Flexural resistance factor for H-Piles or Metal Shells - 1.00

751.36.4 Resistance Factor for Geotechnical Strength Limit States (ϕ_G)

The factored bearing resistance of piles, R_R, shall be taken as:

$$R_R = \phi R_n \quad \text{or} \quad R_R = \phi R_n = \phi_p R_p + \phi_s R_s$$

$$R_p = q_p A_p \quad \text{and} \quad R_s = q_s A_s$$

Where,
R_R = factored bearing resistance (consistent units of load)
R_n = nominal (ultimate) bearing resistance (consistent units of load)
ϕ = resistance factor (skin friction and end bearing)
ϕ_p = resistance factor for end bearing
ϕ_s = resistance factor for skin friction
q_p = unit tip resistance of pile (consistent units of stress)
q_s = unit side resistance of pile (consistent units of stress)
A_p = area of pile tip (consistent units of area)
A_s = surface area of pile side (consistent units of area)

The resistance factors can be obtained depending on the method adopted for design as outlined in the following sub-sections: 751.36.4.1, 751.36.4.2, 751.36.4.3, and 751.36.4.3

751.36.4.1 Ultimate Geotechnical Capacity using Pile Driving Criteria

The Geotechnical Resistance factors are dependent on the energy method used to determine the nominal capacity and the pile driving acceptance criteria during construction.

Method	ϕ
FHWA-modified Gates Formula[1]…………..	0.40
Wave Equation Analysis[2]…………………..	0.50
Dynamic Pile Testing[3,4] on 2% of piles……	0.65

Notes:
1. Gates formula is not considered accurate for pile loading exceeding 600 kips or 300 tons. When pile loading exceeds 600 kips, use wave equation analysis and geotechnical resistance factor of 0.40.
2. WEAP is conducted without pile dynamic measurements or load test, but with field confirmation of hammer performance.
3. See Structural Project Manager or Liaison for use of Dynamic Pile Testing. Dynamic Pile Testing is recommended for projects with friction piles.
4. Dynamic testing requires signal matching, and best estimates of nominal resistance are made from a restrike. Dynamic tests are calibrated to the static load test, when available.

751.36.4.2 Ultimate Geotechnical Capacity using Rational Methods (Axial Compression)

The Geotechnical Resistance factors are dependent on the rational (or static analysis) method used to determine the nominal capacity.

Rational static method to determine nominal capacity	Resistance Factor, ϕ
Skin Friction and End Bearing: Clay and Mixed Soils	
α-method (*Tomlinson, 1987*; *Skempton, 1951*)……………….	0.35
β-method (*Esrig & Kirby, 1979*; *Skempton, 1951*)…………...	0.25
Skin Friction and End Bearing: Sand	
Nordlund/Thurman (*Hannigan et al., 2005*)……………………	0.45
SPT-method *(Meyerhof, 1976)* …………………………………	0.30
CPT-method *(Nottingham & Schmertmann, 1975)* ………….	0.50

Notes:
1. Based on Table 10.5.2.3-1 Resistance Factors for Driven Piles in AASHTO (2009)
2. All methods shown above are described in the FHWA Publication No. FHWA-SA-98-074 (Hannigan, et al. 1997).

751.36.4.3 Ultimate Geotechnical Capacity using Static Pile Load Test(s)

When the nominal resistance has been determined via a successful static load test of at least one pile per site condition without pile dynamic testing, the resistance factor of 0.75 can be used. When dynamic testing is augmented the factor can be increased to 0.80.

751.36.4.4 Ultimate Geotechnical Capacity using Calibrated Resistance Factors

A series of resistance factors were calibrated for steel piles in two geologic regions in Missouri. These regions are: (1) Glaciated Plains and (2) Souteastern Missouri Lowlands. The resistance factors were calibrated based on available pile dynamics tests (PDA/CAPWAP) performed in MoDOT projects. The resistance factors also depend on the method used to estimate the nominal pile capacity. There is no need for a field verification method, since the factors have been calibrated with regional dynamic testing data for the specific pile types listed.

Geologic Region	Pile Type	Design Method	Resistance Factors, ϕ			
			Br.- minor roads	**Br.- major roads**	**Major Br. (<100M)**	**Major Br. (>100M)**
Southeastern Missouri Lowlands	Steel Pipe	DRIVEN	0.58	0.52	0.48	0.46
		Meyerhof	0.37	0.32	0.29	0.27
		Beta-method	0.50	0.45	0.42	0.40
	H-Pile	DRIVEN	0.65	0.60	0.55	0.54
		Meyerhof	0.50	0.41	0.39	0.37
		Beta-method	0.67	0.61	0.57	0.55
Glaciated Plains	Steel Pipe	DRIVEN	0.58	0.53	0.49	0.47
		Meyerhof	0.57	0.51	0.48	0.46
		Beta-method	0.62	0.56	0.53	0.51
	H-Pile	DRIVEN	0.47	0.41	0.38	0.36
		Meyerhof	0.43	0.38	0.35	0.33
		Beta-method	0.61	0.54	0.50	0.47

More detailed information on the calibration of resistance factors is available in the commentary and supplementary document based on MoDOT experience (Kebede, 2010).

751.36.4.5 Ultimate Geotechnical Capacity Uplift Resistance Factors

The recommended values for uplift resistance factors (ϕ_{up}) of single piles are recommended as shown in Table 10.5.5.2.3-1 AASHTO (2009). For group uplift resistance use ϕ_{avg} = 0.50.

751.36.4.6 Evaluation of Group Effects

Group effects for driven piles shall be evaluated as described in this section. Procedures for evaluation of group effects generally involve use of a group efficiency factor, consideration of an “equivalent pier”, or both. Application of the group efficiency factor requires that the nominal resistance for individual piles be multiplied by the factor η to reflect the nominal average capacity of the shaft within a group:

$$R^* = \eta R$$

where

R = nominal resistance of an individual shaft (consistent units of force),

R^* = modified shaft resistance accounting for group effects (consistent units of force), and

η = group efficiency factor established as described in this section.

For pile groups in clay, the nominal axial resistance of the pile group shall be taken as the lesser of:

- The sum of the individual nominal resistances of each pile in the group, or
- The nominal resistance of an equivalent pier consisting of the piles and the block of soil within the area bounded by the piles.

If the cap is not in firm contact with the ground and if the soil at the surface is soft, the individual resistance of each pile shall be multiplied by an efficiency factor η, taken as:

- η = 0.65 for a center-to-center spacing of 2.5 diameters,
- η = 1.0 for a center-to-center spacing of 6.0 diameters.

For intermediate spacings, the value of η may be determined by linear interpolation.

If the cap is in firm contact with the ground, no reduction in efficiency shall be required. If the cap is not in firm contact with the ground and if the soil is stiff, no reduction in efficiency shall be required.

The bearing capacity of pile groups in cohesionless soil shall be the sum of the resistance of all the piles in the group. The efficiency factor, η, shall be 1.0 where the pile cap is or is not in contact with the ground for a center-to-center pile spacing of 2.5 diameters or greater. The resistance factor is the same as that for single piles.

For pile groups in clay or sand, if a pile group is tipped in a strong soil deposit overlying a weak deposit, the block bearing resistance shall be evaluated with consideration to pile group punching as a group into the underlying weaker layer.

751.36.5 Geotechnical Design for Service Limit States

751.36.5.1 Single Piles

In some cases, not common, there will be a need to install a non-redundant single pile, such as in wing walls, small bridges or bents with center-to-center spacing greater than 5 pile diameters. When this is the case the analysis needs to proceed as a single pile. It is recommended that the following methods be followed for these cases.

751.36.5.1.a Empirical Models (Fellenius, 1999)

The load-settlement response of a single driven pile can be estimated using the following relationship:

$$\frac{(q_t')_m}{q_t'} = \left(\frac{\delta}{\delta_u}\right)^g$$

$$\frac{(f_s)_m}{f_s} = \left(\frac{\delta}{\delta_u}\right)^h \leq 1.0$$

Where:

q_t' = unit tip bearing resistance (consistent units of stress)

$(q_t')_m$ = mobilized net unit tip bearing resistance (consistent units of stress)

f_s = unit side-friction resistance (consistent units of stress)

$(f_s)_m$ = mobilized unit side-friction resistance (consistent units of stress)

δ = settlement (consistent units of length)

δ_u = settlement required to mobilize ultimate resistance (consistent units of distance)

$= B/10$ for tip bearing, where B is the pile width or diameter

$= 10mm$ (0.4in) for side friction

g = 0.5 (clay) - 1.0 (sand)

h = 0.02 - 0.5 (may use the average of 0.35)

The elastic compression of the pile itself can also be estimated using:

$$\delta_e = \frac{Q_w z_c}{A_p E_p}$$

Where:

Q_w = load carried by the pile under working load conditions (consistent units of stress)

z_c = distance to the centriod of soil resistance, typically 0.75D (consistent units of length)

A_p = area of cross section of pile (consistent units of area)

E_p = modulus of elasticity of the pile (consistend units of stress)

In the following section other expressions are offered to estimate the compression of the pile.

751.36.5.1b Elastic Settlements

The total settlement of a pile under service limit state loads can be estimated using the Das (2007) and Vesic (1977) approaches and considering three components (1) elastic compression of the pile, (2) settlement due to the load at the pile tip, and (3) settlement of pile caused by the load along the pile shaft. These components should be summed to estimate the total deformation of the pile head.

Elastic compression of the pile = $\dfrac{\left(Q_{wp} + \xi Q_{ws}\right)L}{A_p E_p}$ Das (2007)

Where:

Q_{wp} = load carried by the pile tip under working load conditions (consistent units of load)

Q_{ws} = load carried by the pile shaft (skin) under working load conditions (consistent units of load)

ξ = distribution of the skin friction (0.5 ~ 0.65)

A_p = area of cross section of pile (consistent units of area)

L = embedment length of pile (consistent units of length)

E_p = modulus of elasticity of the pile (consistent units of stress)

Settlement due to the load at the pile tip = $\dfrac{Q_{wp} C_p}{D q_p}$ Vesic (1977)

Where:

Q_{wp} = load carried by the pile tip under working load conditions (consistent units of load)

D = diameter or width of pile (consistent units of length)

q_p = ultimate tip resistance of pile (consistent units of stress)

C_p = empirical constant as defined in the following table.

Type of Soil	C_p	
	Driven Pile	**Bored pile**
Sand (dense to loose)	0.02 – 0.04	0.09 – 0.18
Clay (stiff to soft)	0.02 – 0.03	0.03 – 0.06
Silt (dense to loose)	0.03 – 0.05	0.09 – 0.12

Settlement due to the load along the pile shaft = $\frac{Q_{ws}C_S}{Lq_p}$ Vesic (1977)

Where:

Q_{ws} = load carried by the pile shaft under working load conditions (consistent units of stress)

L = embedment length of pile (consistent units of length)

q_p = ultimate tip resistance of pile (consistent units of stress)

C_s = empirical constant = $\left(0.93+0.16\sqrt{L/D}\right)C_P$

751.36.5.2 Pile Groups

751.36.5.2.a Settlements on SANDS

The settlements of a group of piles under working loads is a complex mechanism, that combines all the superstructure loads and imparts them on to a pile cap and then distributed to the individual piles. Typically, hand calculations are simplified and the gross loads applied to lower layers are estimated as an equivalent footing analogy or using empirical correlations with load tests. However, modern computer analysis techniques are available to model the pile-soil interaction in a group using beam elements and springs or a fully discretized finite element method. Some examples of common software used for these purposes are FB-MultiPier and Group7 for the beam/spring models. For finite element full discretization of soil and structural elements is required, such as in program like PLAXIS and ANSYS. A description of these computer techniques are beyond the scope of these guidelines. Therefore, two simplified close formula solutions are presented herein.

Settlement of pile groups based on SPT (Meyerhof, 1976)

For a pile group in homogeneous sand and not underlain by a more compressible soil layer at a greater depth, settlement of the pile group may be estimated by:

$$\delta_g = \frac{0.96\, q_f \sqrt{B} I_f}{\bar{N}'}$$ …for clean sand $$\delta_g = \frac{1.92\, q_f \sqrt{B} I_f}{\bar{N}'}$$ …for silty sand

Where:

δ_g = estimated total settlement of pile group (mm)

q_f = design working stress (kPa) applied on pile group (design load/group area)

B = width of the pile group (m)

$\bar{N}'$ = average corrected SPT N-value within a depth B below pile toe

I_f = Influence factor for group embedment = $1-\left[D/8B\right] \geq 0.5$

L = embedment length of pile (m)

Settlement based on CPT (Meyerhof, 1976)

To estimate the maximum settlements using CPT results for saturated sands:

$$\delta_g = \frac{42\, q_f\, B\, I_f}{\bar{q}_c}$$

Where:

$\overline{q}_c$ = avg. static cone tip resistance (kPa) within a depth B below the pile tip.
Other terms are defined above.

751.36.5.2.b Settlements in CLAYS

For a pile group consisting of vertical piles, the equivalent footing approach can be used by using an area B x Z that corresponds to the perimeter dimensions of the pile group as shown in Figure 751.36.5.1 below. The bearing pressure is simply the pile group load divided by the area. The load is assumed to spread within a truncated pyramid of side slope at 30 degrees to produce a uniform vertical pressure at the lower levels. The stress at any level is equal to the load carried by the group divide by the projected area at the base of the pyramid. Then, consolidation settlements can be calculated based on the increment in stress in the layers below.

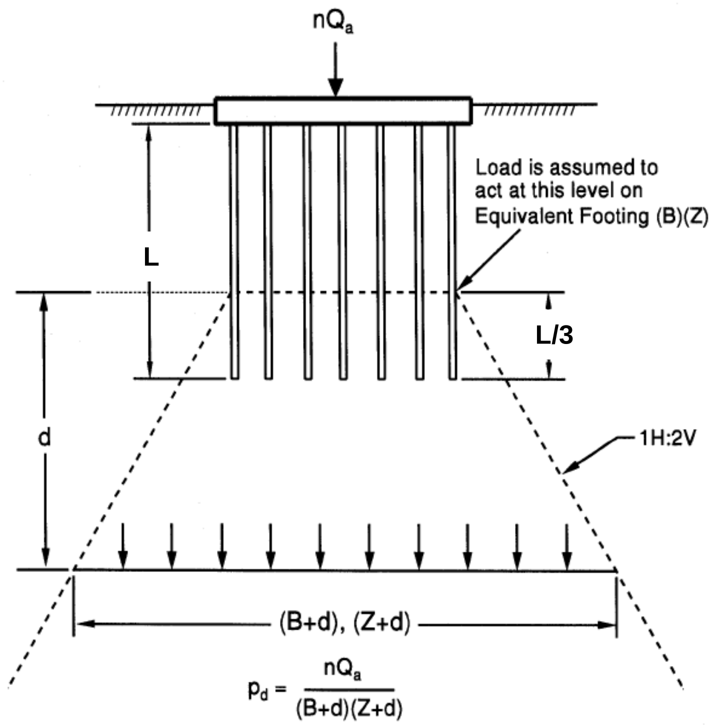

Figure 751.36.5.1 Equivalent footing approach for consolidation settlement calculations.

For calculations of consolidation settlement in clay refer to the procedures outlined in EPG 751.38.4.3. This most likely will require a “Special Foundation Investigation” performed by the Geotechnical Section, which requires undisturbed sampling and determination of compressibility parameters. For compressible layered soils and pile group, refer to Section 10.7.2.3.1 AASHTO (2009).

751.36.6 Structural Design

751.36.6.1 Downdrag & Losses to Geotechnical Strength (kips)

Downdrag, liquefaction and scour all reduce the available skin friction capacity of piles. Downdrag (*DD*) is unique because it not only causes a loss of capacity, but also applies a downward force to the piles. This is usually attributed to embankment settlement. However, downdrag can also be caused by a non-liquefied layer overlying a liquefied layer. Review geotechnical report for downdrag and liquefaction information.

751.36.6.2 Preliminary Structural Nominal Axial Design Capacity (PNDC) of Individual Piles (kips)

The PNDC were calculated with the assumption that the piles are continually braced. This includes the portion of piling that is below ground or confined by solid wall encasement. For portions of piling that are not continually braced, the PNDC must be calculated taking the unbraced length into account.

751.36.6.2.a Steel Piles

$$PNDC = 0.66^{\lambda} F_y A_S$$

Since we are assuming the piles are continuously braced, then $\lambda = 0$. If designing a pile bent structure, scour exists or liquefaction exists then pile shall be checked considering the appropriate unbraced length.

F_y is the yield strength of the pile
A_S is the pile area of steel

751.36.6.2.b Shell Cast In Place Piles (CIP Piles)

$$PNDC = 0.85 f_c' Ac + F_y A_{st}$$

F_y is the yield strength of the pipe pile
A_{st} is the area of the steel pipe (deducting 12.5 % ASTM tolerance and 1/16 inch corrosion where appropriate.)
f_c' is the concrete compressive strength at 28 days
Ac is the area of the concrete inside the pipe pile

Maximum Load during pile driving = $0.90(f_y A_{st})$

Steel Shell is ASTM 252 Grade 2 (35 ksi) or Grade 3 (45 ksi). ASTM 252 allows “the wall thickness at any point shall not be more than 12.5% under the specified nominal wall thickness.” AASHTO recommends deducting 1/16” of the wall thickness due to corrosion. Area of steel shell used in design equations should deduct 12.5% and 1/16” where applicable.

Steel HP Piles					
Section	Area	Structural Nominal Compression Resistance		Structural Factored $(\phi = 0.5)$ Compression resistance	
		$F_y = 36ksi$ (kips)	$F_y = 50ksi$ (kips)	$F_y = 36ksi$ (kips)	$F_y = 50ksi$ (kips)
HP 10x42	12.4	446	620	220	310
HP 12x53	15.5	558	775	275	380
HP 14x73	21.4	770	1070	385	535

CIP Piles						
Diameter	Wall Thickness	A_s (12.5%)	Maximum Driving Resistance Allowed F_y = 35*ksi*	A_s (12.5% & 1/16")	Structural Nominal Axial Compressive Resistance $F_y = 35ksi$ & $f'_c = 4ksi$	Structural Factored $(\phi = 0.6)$ Axial Compressive Resistance $F_y = 35ksi$ & $f'_c = 4ksi$
		in.^{2}	kips	in^2	kips	kips
14	0.25	9.47	300	6.79	720	430
16	0.375	16.15	500	13.12	1075	645

751.36.6.3 Preliminary Factored Nominal Resistance (PFDC) of an Individual Pile (kips)

$PFDC$ = Factored Structural Nominal Resistance – Factored Nominal Downdrag Load

751.36.6.4 Pile Group Layout

Preliminary Number of Piles Required = $\dfrac{Total\ Factored\ Vertical\ Load}{PFDC}$

Layout a pile group that will satisfy the preliminary number of piles required. Calculate the maximum and minimum factored load applied to the outside corner piles assuming the pile cap/footing is perfectly rigid. The general equation is as follows:

Max. Load = $\dfrac{P}{Total\ No.\ of\ Piles} + \dfrac{M_x}{I_x}x + \dfrac{M_y}{I_y}y$

Min. Load = $\dfrac{P}{Total\ No.\ of\ Piles} - \dfrac{M_x}{I_x}x - \dfrac{M_y}{I_y}y$

The maximum factored load per pile must be less than or equal to PFDC for the pile type and size chosen. If not, the pile size must be increased or additional piles must be added to the pile group. Reanalyze until the pile type, size and layout are satisfactory.

The minimum factored load per pile should preferably be greater than zero. If this cannot be practically satisfied, the factored pullout resistance of the pile shall be calculated.

751.36.6.5 Estimate Pile Length and Check Pile Capacity

751.36.6.5.a Estimated Pile Length

Friction Piles:
The estimated pile length will be determined from Pile Soil Interaction (DRIVEN) Analysis. The factor of safety used for this analysis shall be discussed with the appropriate Structural Project Manager or Structural Liaison.

End Bearing Piles:
The estimated pile length is the distance along the pile from the cut-off elevation to the estimated tip elevation considering any penetration into rock. The estimated tip elevation shall not be shown on plans for end bearing piles.

The geotechnical material above the estimated end bearing tip elevation shall be reviewed to review the presence of glacial till or similar layers exist. If these layers are present, then a Pile Soil Interaction (DRIVEN) Analysis shall be performed to verify if pile resistance capacity is reached at a higher elevation due to pile friction capacity.

751.36.6.5.b Check Pile Geotechnical Capacity (Axial Loads Only)

751.36.6.5.c Check Pile Structural Capacity (Combined Axial and Bending)

Structural design checks which include lateral loading and bending shall be accomplished using the appropriate resistance factors.

751.36.6.6 Pile Nominal Axial Compressive Resistance (kips)

The required nominal axial pile compressive resistance must be calculated and shown on the final plans. The factored nominal compressive resistance will be used to verify the pile group layout and loading. The required nominal axial pile compressive resistance will be used in construction field verification methods of nominal axial compressive pile resistance.

Factored Nominal Resistance (FNR)= Maximum Factored Load per Pile

Nominal Axial Compressive Resistance $(Rndr)$

$$Rndr = \frac{Factored\ Nominal\ Resistance}{\phi_G}$$

751.36.6.7 Check Pile Drivability

Practical refusal is defined at 20 blows/inch.

Driving should be terminated immediately once 30 blows/inch is encountered.

If analysis indicated the piles do not have sufficient structural or geotechnical strength or drivability issues exist then consider

- increasing the number of piles
- using higher strength piles

751.36.6.7.a Information to be included on the Plans

Bent No.			
Type			
Kind			
Number			
Approximate Length			
Pile Driving Verification Method			
Design Bearing or Nominal Axial Pile Compressive Resistance			
Minimum Tip Penetration	(*)	(*)	(*)
Criteria For Minimum Tip Penetration			
Pile Standard			
Hammer Energy Required			

751.36.6.7.b Pile Driving Verification Method

- Modified Gates formula
- Dynamic Pile Testing
- Other

751.36.6.7.c Criteria for Minimum Tip Penetration

- Scour
- Tension or uplift capacity
- Lateral stability
- Penetration anticipated soft geotechnical layers
- Minimize post construction settlement
- Minimum embedment into natural ground
- Other

751.36.7 Special Loading Conditions

751.36.7.1 Downdrag Loading

TBD - See AASHTO (2009) for details

751.36.7.2 Lateral Loading

TBD - See AASHTO (2009) for details

751.36.8 References

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C-751.36 Guidelines for Design of Driven Piles - Commentary

C-751.36.1 General

The general design guidelines provide a guide in addition to the engineering judgment that needs to be present in design. For driven piles all pile types may use of both skin and end bearing resistance. Adequate consideration needs to be given when the soil material may not be present along the pile due to scour, pre-drilling or mitigation measures due to downdrag.

C-751.36.2 Steel Pile Characteristics

At this time only steel piles have been considered for the proposed guidelines. However, the used of pre-cast and pre-stressed concrete piles is a viable design alternative. A number of bridges in design-build contracts have included concrete driven piles, which appear to be more economical than steel. Future provision may include concrete piles and more attention should be paid to drivability criteria using the wave equation program.

C-751.36.3 General Design Procedure and Limit States

- Structural Analysis – nothing commented at this time.

- Geotechnical Analysis: The skin friction resistance of driven piles is mobilized before end bearing. This means that the pile skin friction needs to strain some to produce the resistance before it reaches the toe of the pile. However, it is customary to use both skin and end bearing resistance in the analytical methods to estimate pile ultimate capacity and estimate lengths. For H-piling, one can use the perimeter box of the pile to compute the skin friction resistance, which is more conservative as opposed to the explicit perimeter of the H-pile cross section.

- The rational (static analysis) methods used to estimate axial compression pile capacities and design pile lengths should follow the procedures outlined in Hannigan, et al. 1997. Many of the methods are already included in the software program DRIVEN.

- Drivability Analysis: Typically performed with the aid of the wave equation program (WEAP). Additional guidelines should be developed specifically for this procedure if MoDOT will perform in-house or require a specialty contractor to evaluate the drivability as part of the contract requirements. Special provision on Dynamic Methods is already available.

C-751.36.3.1 Design Procedure Outline (*same as provided in EPG*)

1. Determine foundation load effects from the superstructure and substructure for Service, Strength and Extreme Event Limit States.
2. If applicable, determine scour depths, liquefaction information and pile design unbraced length information.
3. Determine if downdrag loadings should be considered.
4. Select preliminary pile size and pile layout.
5. Perform Pile Soil Interaction (DRIVEN) Analysis. Estimate Pile Length and pile capacity.
6. Based on pile type and material, determine Resistance Factors for Structural Strength (ϕs).
7. Determine:
    a. Maximum axial load effects at toe of a single pile
    b. Maximum combined axial & flexural load effects of a single pile
    c. Maximum shear load effect for a single pile
    d. Uplift pile reactions
8. Determine Nominal and Factored Structural Resistance for single pile

   a. Determine Structural Axial Compression Resistance
   b. Determine Structural Flexural Resistance
   c. Determine Structural Combined Axial & Flexural Resistance
   d. Determine Structural Shear Resistance
9. Determine method for pile driving acceptance criteria
10. Determine Resistance Factor for Geotechnical Strength (ϕ_G), based on method of analysis and procedure used following 751.36.4.
11. If other than end bearing pile on rock or shale, determine Nominal Axial Geotechnical Resistance for pile.
12. Determine Factored Axial Geotechnical Resistance for single pile.
13. Determine Nominal pullout resistance if pile uplift reactions exist.
14. Check for pile group effects.
15. Check Drivability of pile using the Wave equation
16. Review Pile Soil Interaction (DRIVEN) Analysis and pile lengths
17. Show proper Pile Data on Plan Sheets.

C-751.36.3.2 Resistance Factor for Structural Strength (ϕ_S)

No commentary developed…

C-751.36.4 Resistance Factor for Geotechnical Strength Limit States (ϕ_G)

The resistance factors to be used in LRFD design procedure are based on the rational (analytical) methods used to estimate initial capacities and pile lengths and the level of effort made in verification of the capacities at the site (dynamic methods, static pile load tests, calibrated from local databases). Sections 751.36.4.1 thru 751.36.4.4 of the EPG are the different approaches to obtain resistance factors for strength limit states depending on the field verification involved during the installation of piles. Typically, the more load tests are performed in the field the higher the resistance factor. Static load tests (sacrificial) are the closest method to obtain the capacity as being experienced by the foundation, however they are costly. Dynamic testing of piles is more efficient as the piles are being tested during the installation. If a certain percentage (2% of production piles) is tested the resistance factor of 0.65 can be used. These resistance factors are applied ot the nominal capacities calculated with the rational static methods. They are primarily used to determine pile lengths for plans.

The design methods are mostly available in the software program DRIVEN which selects the appropriate method for the soil profile for the site. The calibration of resistance factors using the program or method DRIVEN allowed the choice of several methods combined by the program depending on the soil profile. The calibration of resistance factors for the Meyerhof and Beta- methods were carried out on a separate spreadsheet.

C-751.36.5 Geotechnical Design for Serviceability Limit States

Once the capacity of the pile foundations has been determined based on the ultimate limit state, the attention moves to check on the serviceability. Therefore, the predicted performance of the foundations need to be checked under service or working loads (unfactored). The interest in using less conservative ULS resistance factors will require more focus on the anticipated performance of the foundation under service loads. The most common approach is to evaluate the vertical settlement of the piled foundation, some methods are offerred for single pile and pile groups in sections 751.36.5.1 thru 751.36.5.3.

C-751.36.5.1 Single Piles

Settlement of single piles can be estimated using empirical relationships developed by the experience of numerous pile load tests. Fellenius (1999) developed simple expressions to simulate a pile load test for a driven pile in soil. The expressions shown in this section 151.36.5.1.1 are the settlement-load response of a single pile. By preparing a spreadsheet an incremental settlement can be applied and then develop the response for the skin resistance, toe resistance, and elastic compression of the pile. This way a load vs. settlement response curve can be developed and the working load determined based on the allowable settlement for the foundation or vice versa.

Das (2007) and Vesic (1977) developed simple expressions to estimate the elastic response of a single pile under a working load. Das (2007) proposed the expression to estimate the elastic compression of the structural pile member. It is important to recall that settlements for pile foundations are anticipated to be very small and the elastic compression of the structural pile will be a significant contribution that needs to be taken into account. These expressions have different empirical constants for the general soil type where the pile was driven into.

C-751.36.5.2 Pile Groups

The settlements of a group of piles under working loads is a complex mechanism, that combines all the superstructure loads and imparts them on to a pile cap and then distributed to the individual piles. Typically, hand calculations are simplified and the gross loads applied to lower layers and estimated as an equivalent footing analogy or using empirical correlations with load tests. However, modern computer analysis techniques are available to model the pile-soil interaction in a group using beam elements and springs or a fully discretized finite element method. Some examples of common software used for these purposes are Multi-FB-Pier and Group7 for the beam/spring models. For finite element full discretization of soil and structural elements is required, such as in program like PLAXIS and ANSYS. A description of these computer techniques are beyond the scope of these guidelines. Therefore, two simplified close formula solutions were presented in the guidelines.

Two closed formula solutions developed by Meyerhof (1976) are available for pile groups installed into SAND. The first one is for clean and silty sands that have been characterized by geotechnical engineers using the standard penetration test (SPT) technique. The corrected N' value is necessary to perform the analysis. This correction is the overburden correction and normalized to a standard energy of 60% efficiency. The second solution is used when cone penetrometer test (CPT) data is available from the geotechnical explorations. An average cone tip resistance, q_c, is used to estimate the settlement of the pile group.

For a pile group consisting of vertical piles, the equivalent footing approach can be used by using an area B x Z that corresponds to the perimeter dimensions of the pile group in plan view. The bearing pressure is simply the pile group working load divided by the area. The load is assumed to spread within a truncated pyramid of side slope at 30 degrees to produce a uniform vertical pressure at the lower levels. The stress at any level is equal to the load carried by the group divide by the projected area at the base of the pyramid. Then, consolidation settlements can be calculated based on the increment in stress in the layers below. It is recommended that a geotechnical engineer carry out these calculations, since a "special investigation" should be performed then compressible soils are anticipated under the pile group. Interpretation of laboratory data to determine the geotechnical consolidation parameters is required.

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