Center for Transportation Infrastructure and Safety

Functionally Graded Biomimetic Energy Absorption Concept Development for Transportation Systems

by

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Technical Report Documentation Page

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16. Abstract

Mechanics of a functionally graded cylinder subject to static or dynamic axial loading is considered, including a potential application as energy absorber. The mass density and stiffness are power functions of the radial coordinate as may be the case with variable-density open-cell or closed-cell foams. Exact solutions are obtained in the static problem and in the case where the applied load is a periodic function of time. The absorption of energy is analyzed in the static problem that is reduced to an easily implementable nondimensional formulation. It is demonstrated that by using foams with a variable in the radial direction mass density it is possible to achieve higher energy absorption while at the same time saving weight.

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1. Description of the problem

Functionally graded materials have been analyzed in a variety of applications for the last three decades (e.g., V. Birman and L. W. Byrd, "Modeling and analysis of functionally graded materials and structures," *Applied Mechanics Reviews*, vol. 60, pp. 195-216, 2007). These materials are invariably composite where variations of the content, architecture or orientation of one of the constituent phases provide a desirable effect. In particular, functionally graded shells may provide numerous advantages under static, dynamic and environmental loads compared to their homogeneous (not graded) counterparts. The pioneering work on the stress problem in cylinders with longitudinally and radially varying properties was published by Lekhnitskii in his classical monograph, “Theory of Elasticity of an Anisotropic Body.” Functional grading of materials can be used to achieve a variety of goals, including alleviation of residual stresses, reducing stresses during lifetime of the structure, improvement of stability and dynamic response, preventing fracture and fatigue, etc. One of intriguing and so far unexplored possibilities is related to using low-stiffness regions of a graded structure for enhanced energy absorption. Such attempt is motivated by the observation that materials with a reduced stiffness possess higher toughness. This phenomenon has been documented for both engineering and biological materials (Fig. 1). However, there are limits to a possible stiffness reduction aimed at achieving higher resilience and toughness that are superimposed by strength requirements (Fig. 2). Accordingly, the problem of energy absorption and toughness of functionally graded materials should be considered in conjunction with the stress problem including the distribution of local stresses and the check of local strength.

The problem of energy absorption by engineering materials has been considered in numerous studies. Examples are found in recent reviews, such as a comparison of various energy absorbing materials (P. Qiao, M. Yang, and F. Bobaru, "Impact mechanics and high-energy absorbing materials: Review," *Journal of Aerospace Engineering*, vol. 21, pp. 235-248, 2008), as well as research of peculiar energy absorption mechanisms and effects in particular material systems including nanocomposites (L. Sun, R. F. Gibson, F. Gordaninejad, and J. Suhr, "Energy absorption capability of nanocomposites: A review," *Composites Science and Technology*, vol. 69, pp. 2392-2409, 2009), various foams (U. K. Chakravarty, "An investigation on the dynamic response of polymeric, metallic, and biomaterial foams," *Composite Structures*, vol. 92, pp. 2339-2344, 2010), composite materials under axial loading (S. T. W. Lau, M. R. Said, and M. Y.

Yaakob, "On the effect of geometrical designs and failure modes in composite axial crushing: A literature review," *Composite Structures*, vol. 94, pp. 803-812, 2012), and ballistic helmets (S. G. Kulkarni, X. L. Gao, S. E. Horner, J. Q. Zheng, and N. V. David, "Ballistic helmets – Their design, materials, and performance against traumatic brain injury," *Composite Structures*, vol. 101, pp. 313-331, 2013).

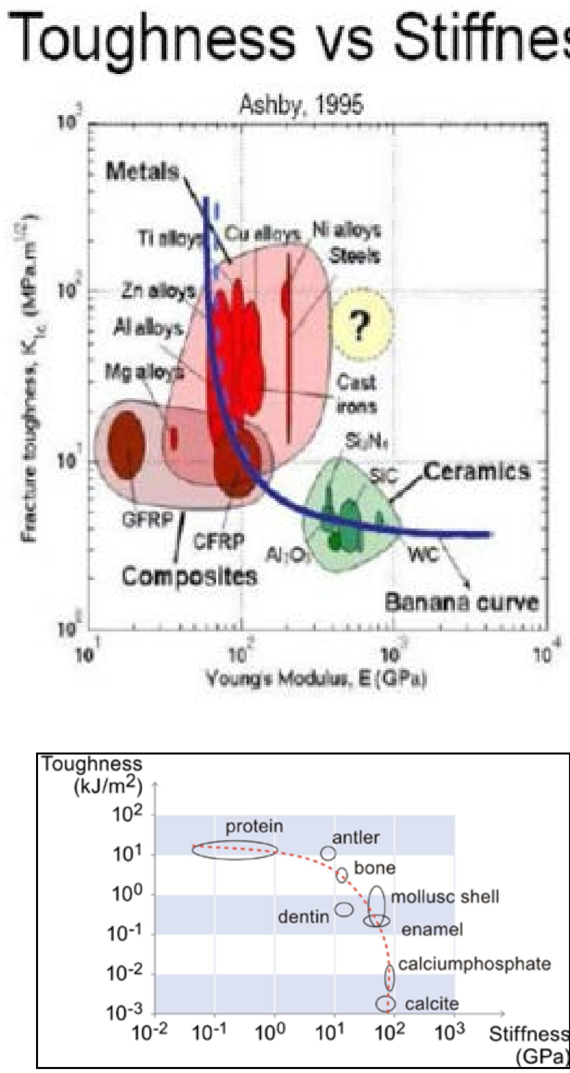

Fig. 1. Toughness of engineering (top figure) and biological (bottom figure) materials increases as their stiffness decreases. Accordingly, it is concluded that energy absorption increases in compliant materials.

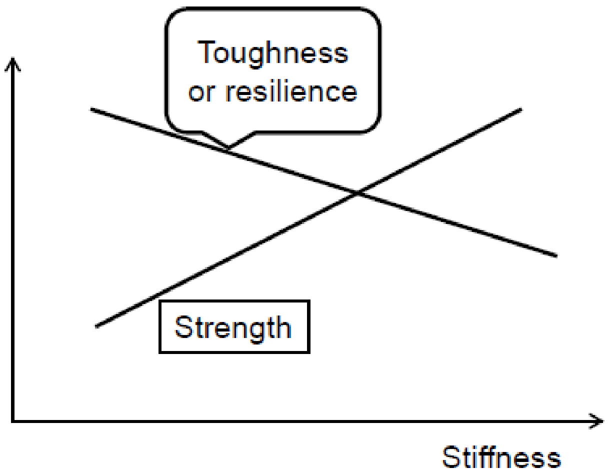

Fig. 2. Dual effect of higher stiffness: the strength of stiffer materials is usually higher, but their toughness and resilience are lower.

Foams represent an example of highly efficient energy absorbing materials (Fig. 3). An example of a recent foam application concept aimed at maximum energy absorption is its use in personal armor. The stiffness and energy absorption capacity of foam depend on its density and architecture (i.e., open-cell vs. closed-cell foams). The beneficial effect of a closed-cell polyurethane foam layer on the blast response of sandwich panels was investigated (Y. A. Bahei-El-Din and G. J. Dvorak, "Behavior of sandwich plates reinforced with polyurethane/polyurea interlayers under blast loads," *Journal of Sandwich Structures and Materials,* vol. 9, pp. 261-281, 2007). The designs considered in this research utilized a solid polyurethane layer and a polyurethane foam layer between the outer facing and core. The impact energy being partially absorbed by these layers, the authors achieved a reduction in the peak kinetic energy and core compression by almost 50%. The study by the same authors utilizing a similar approach with either solid polyurethane or polyurethane foam interlayer between the outer facing and core also demonstrated their effectiveness in the mitigation of the effect of a low velocity impact. In general, foams present a designer with the desirable flexibility enabling him to utilize spatial variations of the density, strength and stiffness, tailoring the response of the structure to specific design requirements.

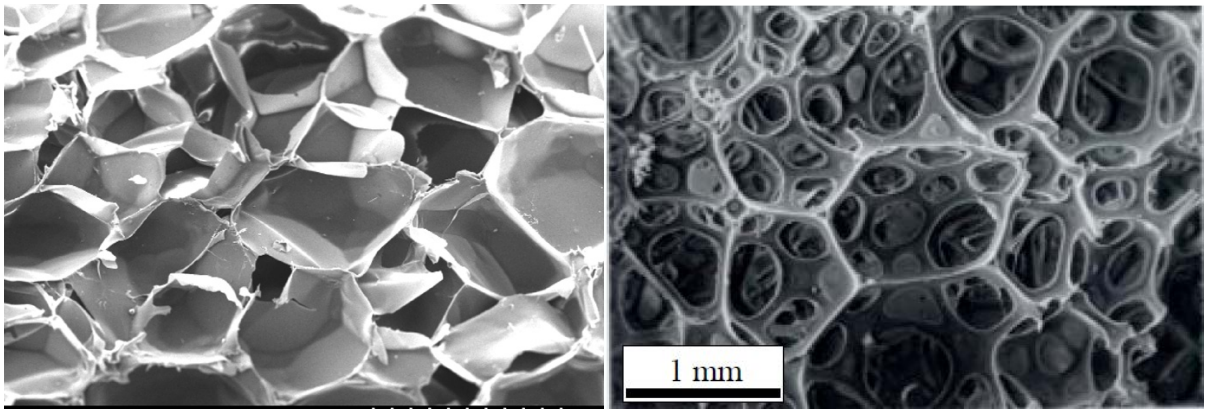

Fig. 3. Examples of polyurethane foam (From Kaushiva B., “Structure-property relationships of flexible polyurethane foams,” Virginia Polytechnic Institute and State University, Blacksburg, 1999 and Marsavina L, Sadowski T, Knec M, Negru R., “Non-linear behaviour of foams under static and impact three point bending,” International Journal of Non-Linear Mechanics 45 (10):969-975, 2010). High toughness at ultimate loading is achieved through dissipation of energy in the phase corresponding to buckling of truss and walls of foam material and the subsequent solidification. In case of multiple loading events, buckling and solidification are not permitted but significant amounts of energy are still dissipated due to low stiffness of the foam structure prior to the onset of failure.

Besides the previously mentioned work of Lekhnitskii, functionally graded cylinders under axial loading have been considered in both static and dynamic applications. In these studies the cylinders were characterized by various theories of shells downplaying the three-dimensional nature of the problem. Contrary to the efforts utilizing shell theories, the present problem is solved using a three-dimensional formulation of the theory of elasticity, expanding the approach of Lekhnitskii without limitations on the thickness of the cylinder. In addition to the solution of the static and dynamic stress problems, the study demonstrates the absorption of energy under static compression facilitated by grading the material density in the radial direction.

Note that an attempt to maximize the energy absorption by using a functionally graded material with a strategically reduced local stiffness can be traced to biology. For example, the biological enthesis (tendon-to-bone insertion site) is characterized by functional grading and a dip in the stiffness between tendon and bone. The research of the enthesis leads to the assumption that such distribution of properties is conducive to higher resilience of the enthesis (e.g., Y. X. Liu, S. Thomopoulos, V. Birman, J. S. Li, and G. M. Genin, "Bi-material attachment through a compliant interfacial system at the tendon-to-bone insertion site," Mechanics of

Materials, vol. 44, pp. 83-92, 2012). While contrary to the case of enthesis in the problem considered in the paper the material is graded in the direction perpendicular to the load rather than along the load, the analogy with natural biological structures deserves an acknowledgement.

2. Approach and methodology

Consider a cylindrical shell of the inner and outer radii $r = a$ and $r = b$, respectively, and the length L that is supported at one end and loaded by an axial static or dynamic force at the opposite end (Fig. 4). The loaded end of the shell $(z = L)$ is not restrained. The supported end $(z = 0)$ is prevented from axial displacements but free to expand in the radial direction, so that it does not constrain radial deformations of the shell. The shell is manufactured from a quasi-isotropic material that is functionally graded in the radial direction so that the properties, including the stress tensor, vary with the radial coordinate, but the material remains locally isotropic at every point. This implies grading on the macroscopic scale where the mass density is a slowly changing function of the radial coordinate.

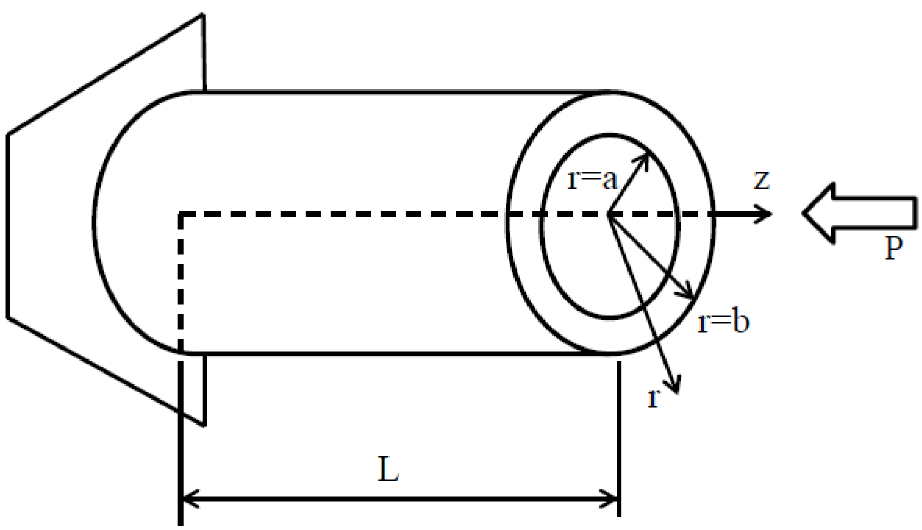

Fig. 4. Formulation of the problem: Foam cylinder between constrained within rigid enclosure formed by inner and outer walls protecting foam from external side loads and environment is subject to axial loading. By varying the mass density and stiffness from the inner to outer radius significant energy can be absorbed by a relatively light structure.

The analysis is conducted by the following assumptions:

1. The state of generalized plane strain prevails throughout the cylinder, i.e. each cross section perpendicular to the axis of the cylinder remains plane during deformation, without warping.
2. The problem is both physically and geometrically linear.
3. The cylinder remains stable, i.e. we concentrate on the stress and energy absorption analyses, without concerns related to structural stability.

The second assumption implies that even for low-density sections of the material, deformations are small and the stress-strain relationships are linear elastic. In the study, we consider an example of open-cell foam that possesses a nonlinear response in the large strain range that is typical for some of energy absorption applications. However, if this material is employed in the structure experiencing multiple repeated or periodic loads, the branches of the response corresponding to buckling of struts of the foam (elastic collapse) and subsequent densification (plastic collapse) cannot be allowed (Fig. 5). Accordingly, the application of foam would be limited to strains corresponding to the linear elastic response with the requirement that the foam should not collapse, while absorbing the maximum possible amount of energy during deformation.

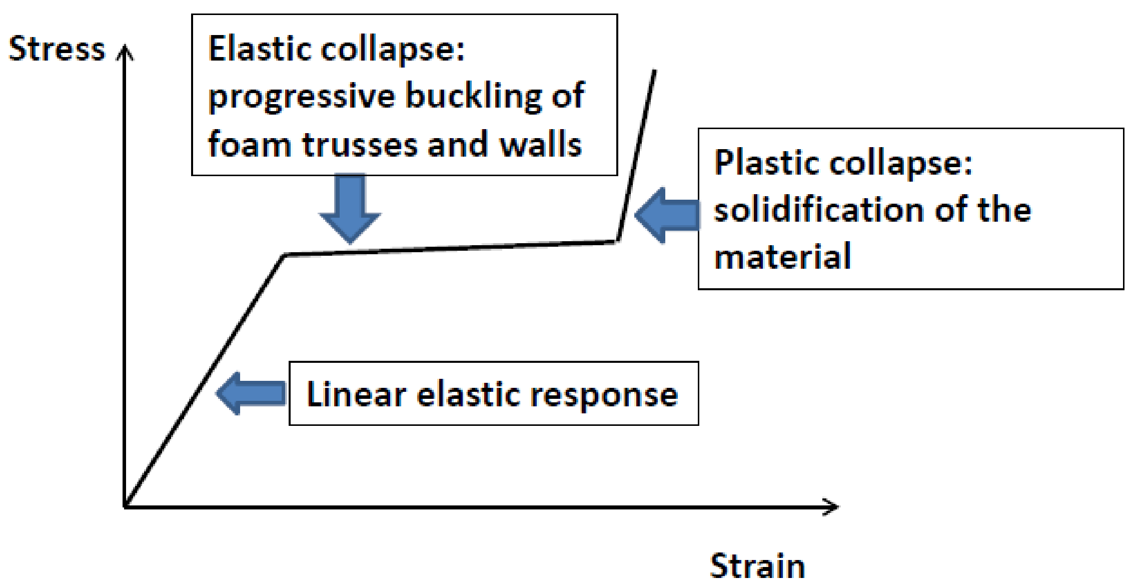

Fig. 5. Stress-strain relationships of typical foam. Absorbed energy is the area below the stress on the stress-strain diagram.

The approach to the comprehensive analysis of the energy absorbent functionally graded cylindrical foam structure includes the following phases:

- Micromechanics, i.e. the derivation of the variations of the tensor of stiffness as a function of the spatial distribution of the foam density;
- Macromechanics, i.e. the exact solution of the equations of linear theory of elasticity yielding stresses throughout the structure;
- Energy absorption analysis;
- Optimization, i.e. the development of the cylinder with the maximum energy absorption and minimum weight, while preserving the strength throughout the structure.

Based on the available data, it is realized that the optimum mass density distribution for maintaining the required strength, while absorbing the maximum energy, corresponds to the stiffness gradually decreasing from the inner to outer radius of the cylinder. The stiffness of open-cell and closed-cell foams being a power function of the mass density, the micromechanical problem is addressed accordingly using dense and stiffer foam close to the inner surface and more compliant and less dense foam in the vicinity to the outer surface. Both the mass density and the tensor of stiffness considered in the study vary through the thickness according to power laws being functions of the radial coordinate (powers are different for the stiffness and mass density).

2.1 Detailed Methodology

In this section we describe the methodology of the solution in detail. Mathematical details implementing the described solution are provided in the forthcoming article in the ***International Journal of Engineering Science***.

2.1.a. *Micromechanics* is a theoretical formulation that provides the properties of the material on in terms of the properties and architecture of the constituent phases. Sometimes this step is avoided in structural papers on functionally graded materials by arbitrary assuming the spatial distribution of the properties. Such approach is fundamentally mistaken since it violates the micromechanical aspect of the problem (arbitrary selection of the material properties may not be reproducible by any feasible mixture of the constituent materials) and potentially overlooks fundamental thermomechanical principles.

In this study, we presume that the mass density of foam varies along the radial coordinate r according to a power law:

$$\rho = \underset{\sim}{k}^{1/2} r^{m/2} \tag{1}$$

where $\underset{\sim}{k}$ is a coefficient (the dimensions of this coefficient should be such that the units of the expression in the right side of Eqn. (1) comply with the units of mass density) and m is a power.

The tensor of stiffness of foams is related to the mass density by a power law as evidenced by experimental data (e.g., Fig. 6). Varying the density according to (1), this tensor can be evaluated as a power-law function of the radial coordinate. This evaluation accomplishes the micromechanical phase of the analysis.

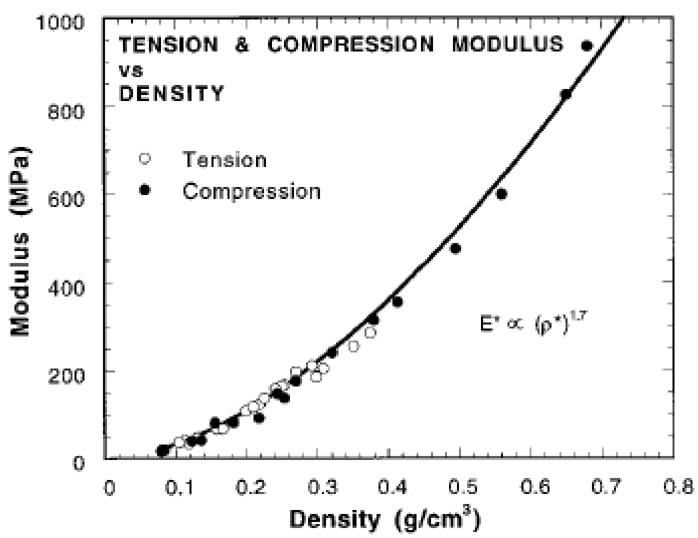

Fig. 6. Variations of the modulus of elasticity of typical foam with density. In the figure, the modulus of the closed cell foam varies as the 1.7 power of mass density. From S.H. Goods, C.L. Neuschwanger, L.L. Whinnery and W.D. Nix, "Mechanical Properties of a Particle-Strengthened Polyurethane Foam," Journal of Applied Polymer Science, Vol. 74, pp. 2724-2736, 1999. Similar results have been published for open-cell foams, though the power was different.

One issue that has to be considered simultaneously with the stress analysis is related to the variation of the strength of foams with the radial coordinate. This knowledge is necessary to check the strength at every location and avoid elastic collapse throughout the cylinder (maintaining strength under design load is necessary if the structure is undergoing repeated loads). For example, for open-cell foams, according to (L. J. Gibson and M. F. Ashby, *Cellular* *solids, structure and properties*, 2nd ed. Cambridge: Cambridge University Press, 1997), the elastic collapse stress is given by

$$\sigma_z^{collapse} = 0.05 E_s \left(\frac{\rho}{\rho_s} \right)^2 \tag{2}$$

where E_s and ρ_s are the elastic modulus and mass density of the solid foam material, respectively. Accordingly, it is possible to simultaneously trace both the variations of the tensor of stiffness as well as the strength of foam throughout the cylinder for a prescribed variation of the mass density in the radial direction.

2.1.b. Macromechanics

In the axisymmetric formulation, the equation of equilibrium in the axial direction is

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \tag{3}$$

where σ_r and σ_θ are radial and circumferential stresses, respectively.

Upon the substitution of the constitutive relations and the strains expressed in terms of the radial displacement this equation is transformed to a heterogeneous Bessel equation in terms of the radial displacement and the axial strain:

$$\frac{d^2u}{dr^2} + \frac{1+m}{r}\frac{du}{dr} - \frac{1 - {mn}/{k}}{r^2} u = -\frac{mn}{kr}\varepsilon_z \tag{4}$$

where k is a coefficient dependent on the material properties and distribution that can be solved in terms of ε_z.

The subsequent solution is obtained satisfying the boundary conditions at the inner and outer surfaces of the cylinder and the condition of axial equilibrium at large (i.e., the applied force is equal to the integral of axial stresses in the foam over the cross sectional area of the cylinder). The result is the applied force vs. axial strain relationship that can be used to evaluate axial strain ε_z that is subsequently employed to determine axial and radial displacements and stresses throughout the cylinder.

In the dynamic problem the study concentrated on the case where the cylinder is loaded by a periodic-in-time harmonic axial force. Upon lengthy transformations, the equation of motion in terms of the amplitudes of axial (U) and radial displacements (W) is reduced to the form

$$\frac{\partial^2 U}{\partial r^2}+\frac{1+m}{r}\frac{\partial U}{\partial r}-\left(\frac{1-{mn}/{k}}{r^2}-\frac{S\omega^2}{r^{m/2}}\right)U=-\frac{mn}{kr}\frac{\partial W}{\partial z} \tag{5}$$

where S is a constant coefficient dependent on material and geometry and ω is a frequency of the driving force.

The solution of the nonhomogeneous equation (5) is obtained in terms of Bessel functions by the method of variation of parameters (Lagrange's method). Two constants of integration are specified from the boundary conditions on the cylinder surfaces. Subsequently, the dynamic stresses can be evaluated throughout the cylinder.

2.1.c. Energy absorption analysis

The external force being applied to the cylinder as shown in Fig. 1, the absorbed energy in the static problem can be obtained as a product of this force and the axial displacement of the loaded end. In the dynamic problem, the solution is more complicated. This is because the loss factor in the foam depends on the mass density. Accordingly, this factor varies with the radial coordinate. If the loss factor can be expressed in terms of the mass density of foam, the previously discussed dynamic analysis can be applied to the evaluation of energy dissipation during a cycle of forced motion since the solutions for both axial and radial vibrations are known at every point of the cylinder. Thus, the overall energy dissipation per cycle can be calculated integrating the local energy dissipation density over the volume of the cylinder.

2.1.d. Optimization

It is possible and desirable to optimize the cylinder. First of all, the analysis of the increase of the energy absorption achieved by reducing the mass density of the foam has to be conducted under the constraint related to retaining its strength. As could easily be predicted, the strength of the foam decreases if its density is reduced. Accordingly, a comprehensive optimization analysis requires the following steps:

- Choice of the variation of mass density of foam in the radial direction;
- Specifying the corresponding weight of the structure;

- Evaluation of the tensor of stiffness and the strength (elastic collapse stress) as functions of the radial coordinate, from the inner to outer surface;
- Computation of stresses throughout the radial coordinate;
- Check of the strength throughout the radial coordinate;
- Evaluation of the energy absorption.

If the strength is violated at any location and the structure can be subject to repeated loads, the density distribution should be adjusted and the optimization process repeated. If the structure is capable of carrying the applied load, it may be possible to improve it modifying the mass density distribution so that the weight is reduced. The optimization cycle for the correspondingly modified cylinder would be repeated to check that the required amount of energy is absorbed and the strength is retained. Based on our numerical analysis, it is predicted that more efficient designs will have the mass density decreasing from the inner to outer surface. The strength of such designs is also superior, the inner sections with higher density and strength being subject to higher stresses than less stiff sections close to the outer surface that carry lower load and have smaller strength.

3. Findings

The effects of the ratio of inner-to-outer radii ratio and functional grading on the absorbed energy and weight of the cylinder are demonstrated in Fig. 7 where $i_1 = 0.7$, $i_2 = 0.5$ and $i_3 = 0.6$. The following notations are used:

$$i_1 = \frac{\rho(\rho = a)}{\rho_s}, \qquad i_2 = \frac{\rho(\rho = b)}{\rho_s} \tag{6}$$

in the graded cylinder and $i_3 = \frac{\rho}{\rho_s} = 0.6$ in the homogeneous cylinder.

As is observed in this figure, both the energy absorption and weight savings improve in a relatively "thicker" shell, i.e. at smaller ratios a/b. A sharper grading of the material investigated in additional examples included in the forthcoming journal paper further improves the energy absorption and marginally decreases the weight of the cylinder. Extreme grading cases have not been considered since they would likely result in warping of the cross sections perpendicular to the cylinder axis violating the plane strain assumption adopted for the present analysis.

Furthermore, a very low local material density in a part of the cross section would cause the loss of strength even at moderate loads.

The effect of the material density at the inner radius on the energy absorption is further elucidated in Fig. 8 for the inner-to-outer surface radii ratio equal to 0.5. While the mass density at the inner radius varies, the density at the outer radius is kept constant. A higher density in the vicinity to the inner radius of the cylinder is detrimental both reducing the energy absorption and increasing weight. This could be anticipated since a stiffer inner section of the cylinder corresponds to the overall higher stiffness as long as the assumption of plane strain is accepted. The change of the inner-to-outer radii ratio considered in examples included in the forthcoming journal paper does not alter the conclusion.

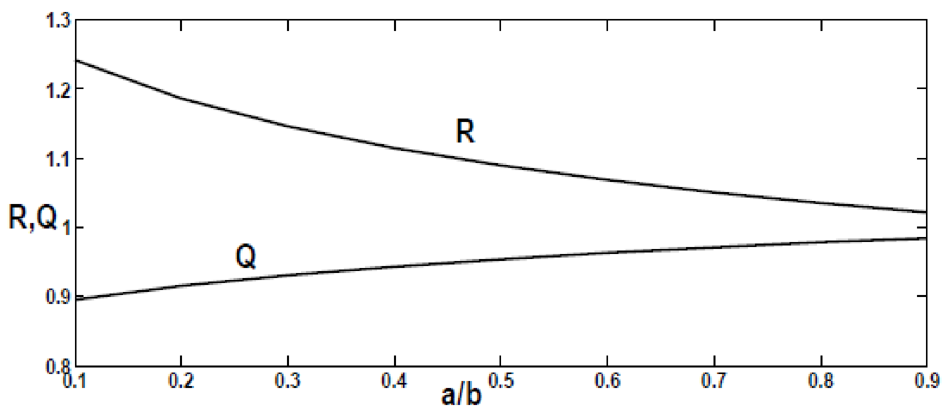

Fig. 7. Energy absorption (R) and weight (Q) in a functionally graded cylinder ($i_1 = 0.7,\ i_2 = 0.5$) normalized by the energy absorption and weight in a homogeneous cylinder ($i_3 = 0.6$) as functions of the inner-to-outer radii ratio.

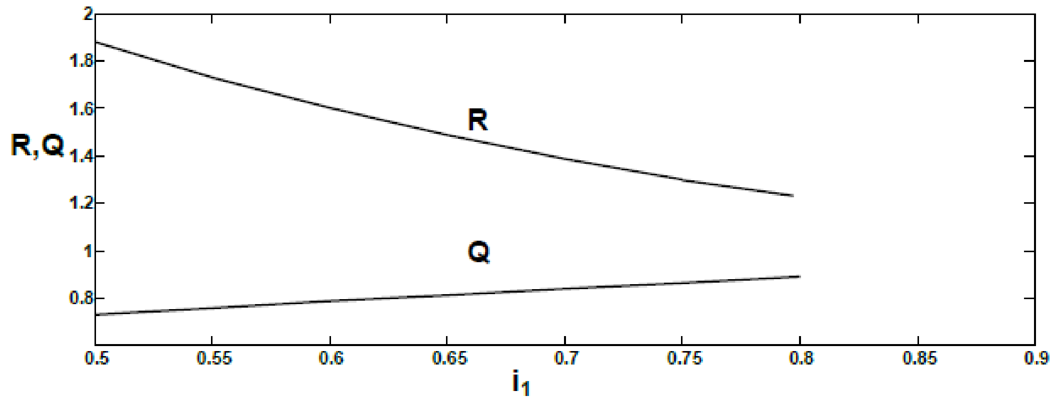

Fig. 8. Effect of the material density at the inner surface on the energy absorption (R) and weight (Q) for $a/b = 0.5$. The mass density at the outer surface is kept constant $i_2 = 0.4$. The energy absorption and weight are normalized by those in a homogeneous cylinder with $i_3 = 0.6$.

4. Conclusions

Exact solutions to the problems of stress and displacement distribution and energy absorption in a functionally graded cylinder subject to a static axial force are obtained in the case where the stiffness varies as a power function of the radial coordinate. It is shown that such grading is achievable if the cylinder is manufactured from open-cell or closed-cell foam. The dynamic response of the cylinder is also considered in the case where it is loaded by a periodic-in-time harmonic force; the exact solution is developed for this dynamic case. Numerical results are demonstrated for the energy absorption in the static case. It is shown that a relatively higher energy absorption and weight savings over a homogeneous cylinder are possible in a functionally graded cylinder with a smaller inner-to-outer radii ratio. An overall reduction of the stiffness throughout the radial coordinate is beneficial to the energy absorption as well as resulting in a lighter cylinder. Such stiffness reduction may be limited by the strength considerations that should be included in the analysis using the developed solution. Accordingly, if the main function of the cylinder is energy absorption, the problem of optimum functional grading should be considered accounting for the stress and strength distribution throughout the graded material.

The present research provides the complete analytical tool to designers developing energy and shock absorption devices using open-cell and/or closed-cell foam cylinders. The variable-density cylindrical foam devices analyzed in this research are light and can be used on a repeated basis as long as the foam remains intact, operating in the linear elastic phase.

Furthermore, the availability of the developed structures for energy absorption in extreme cases of overloading utilizing huge energy absorbing capacity of foam in the nonlinear range corresponding to buckling of cell trusses and walls can be analyzed extrapolating the developed analytical methodology.

Using functionally graded variable density foams in shock and energy absorption applications in transportation systems is feasible; manufacturing aspects can be addressed using modern production methodologies. Besides polymeric foams, metallic foams can also be incorporated in such applications providing higher strength and versatility. Further study of the proposed concepts, extrapolating the range of loading and geometries would be fully justified.

In conclusion, using variable mass density foams in structures for enhanced energy absorption represents a promising concept. The application of such concept to transportation systems may improve safety and reliability of such systems without increasing their weight or cost.

5. Recommendations

The proposed concept of the functionally graded energy and/or shock absorber has been analytically proven and justified on the example of a variable mass density foam cylinder. The concept can further be extended and applied to foam energy absorbers of various shapes and purposes. The principle of such absorbers is concentrating low-density foam in the regions where it can dissipate larger amounts of energy, without sacrificing strength. Typically, the compromise between the desirable compliance and necessary strength can be achieved in relatively less stressed areas of the structure. Two types of foam absorbers that can be developed include absorbers designed to withstand repeated loads and those for single event energy absorption. The former operate in the linear elastic range, avoiding elastic collapse associated with irreversible buckling of foam trusses and walls. The latter can utilize energy absorption up to ultimate failure occurring with the solidification of the foam.

The practical implementation of the developed concept requires a number of experiments, dependent on the particular foam (closed-cell vs. open-cell; polymeric vs. metallic, etc.). This includes the confirmation of the coefficients and powers in the power laws relating the stiffness of the foam to the mass density for a particular material and to the type of foam chosen for design. Also, the relationship between the strength (elastic collapse stress) and mass density should be verified. While there is significant amount of information for the stiffness vs. mass density relationships of various foams, mostly generated by Ashby and Gibson, the strength vs. density data is very limited. Additional data that can be useful for the implementation of the concept into design includes a possible shape optimization of the foam structure. If foam is used for the energy absorption in dynamic problems, it becomes critical to experimentally determine the loss factor of foam in the broadest possible range of its mass densities. Such information is unavailable today and its generation would be paramount to a development of foam-based functionally graded energy absorbers.

If the concept is implemented using metallic foams, such factors as viscoelasticity and plastic effects may be considered for the improvement of efficiency. In polymeric and even in metallic foams an interesting option would be to “fill” foam with a “soft” material that would not significantly increase the load-carrying capacity and weight, but improve the energy absorption capability.

6. Meeting the proposed deliverables in the research project

The proposed deliverables included presentations at professional meetings and a paper in an archival journal demonstrating the concept, theoretical formulation and solution to the problem of energy absorption in transportation systems using functionally graded foam cylinders. The effort was completed on December 31st, 2013, and the relevant paper fully describing the study has been accepted for publication in the ***International Journal of Engineering Sciences*** (the electronic version of the paper is scheduled for publication on March 31, 2014 followed with the paper version). The results of the study will be presented at the ***2014 International Mechanical Engineering Congress and Exposition*** (IMECE 2014) as an invited paper in the symposium organized by Professor Xin-Lin Gao, University of Texas at Dallas (invitation received and the presentation has been confirmed by PI). The paper that was motivated by the accomplished study extrapolating the proposed methodology to functionally graded spherical shells with the application to protective helmets is presently in preparation.