PRESERVATION OF MISSOURI TRANSPORTATION INFRASTRUCTURES

VOL I: Bridge Design & Load Rating

VALIDATION OF FRP COMPOSITE TECHNOLOGY
THROUGH FIELD TESTING

Strengthening of Bridge P-0962
Dallas County, MO

Prepared for:
Missouri Department of Transportation
University of Missouri-Rolla
(Project Code R03MO4-1)

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A. INTRODUCTION

A.1 General Description

In the following report, the analysis and design procedures used in the upgrade of the load-posted Bridge P-0962, located in Dallas County, MO are summarized. Figure 1 shows a picture of the bridge. The total bridge length is *127.5 ft* and the total width of the deck is *23.75 ft*.

Figure 1 – Bridge P-0962

The structure has three spans and each of them consists of three reinforced concrete (RC) girders monolithically cast with a *6 in.* slab, as depicted in Figure 2. Each span is provided with one transversal beam. All spans are *42.5 ft* long.

Figure 2 – Superstructure of the Bridge

A.2 Objectives

The objective of this document is to provide an analysis of the structure and the design calculations for its strengthening using externally bonded fiber-reinforced polymer (FRP) systems and steel reinforced polymer (SRP) systems. The FRP systems consist of FRP laminates to be installed by manual lay-up and FRP bars to be near surface mounted (NSM). The SRP system consist of SRP laminates installed by manual lay-up.

A.3 Assumptions

The following assumptions are made:

a) Nominal material properties for steel and concrete. At the onset of the project, existing material properties were validated in the field by extracting two concrete cores and steel bar sample. The resulting values are: $f`_c=6845\ psi$, and $f_y=40\ ksi$.
b) Load configurations and analysis are consistent with AASHTO[1] Specifications; and
c) Design of the strengthening system is in compliance with ACI 440.2R-02[2] where applicable.

B. STRUCTURAL ANALYSIS

B.1 Load Combinations

For the structural analysis of the bridge, definitions of the design truck, load lane, and design lane are necessary, as well as the transversal load distribution. These issues will be discussed in the next two sections. A plan view of the bridge piers and abutments is shown in Figure 3.

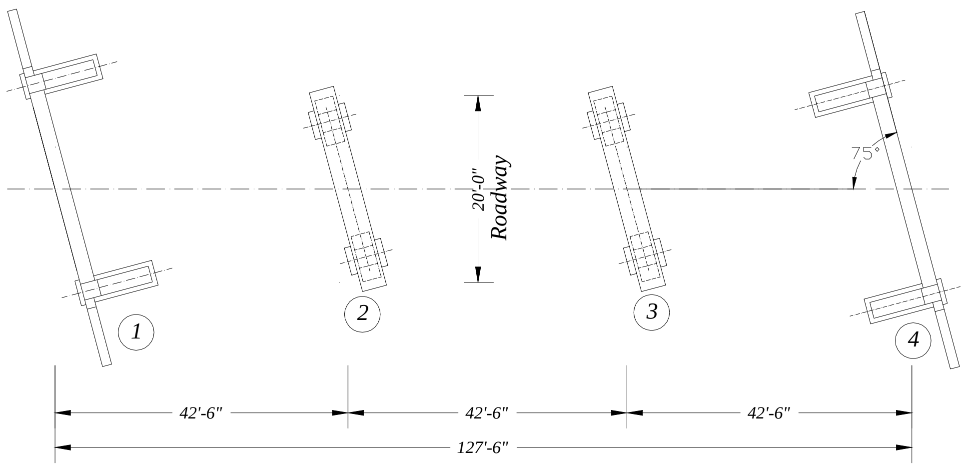


Figure 3 – Plan View of the Bridge (Not to scale)

Ultimate values of bending moment and shear force are obtained by multiplying their nominal values by the dead and live load factors and by the impact factor according to AASHTO Specifications as shown in Eq. (1):

$$\omega_u = 1.3\left[\beta_d D + 1.67(L+I)\right] \tag{1}$$

where D is the dead load, L is the live load, $\beta_d=1.0$ as per AASHTO Table 3.22.1A, and I is the live load impact calculated as follows:

$$I = \frac{50}{L+125} \tag{2}$$

and L represents the span length from center-to-center of support. The impact factor can be assumed equal to *1.29* for all spans.

B.2 Design Trucks, Load Lanes and Design Lanes

The analysis of the bridge is carried out for an HS20-44 truck load (which represents the AASHTO design truck load) and for a 3S2 truck load as requested by MoDOT, having geometrical characteristics and weight properties as shown in Figure 4.
According to AASHTO Section 3.6.3, roadway widths between *20.0* and *24.0 ft* shall have two design lanes, each equal to one-half of the roadway width. Although the roadway width of the bridge is *20.0 ft*, only one design lane has been considered for the truck load analysis. For the load lane analysis, however, two or one lanes will be considered depending on the worst loading scenario.

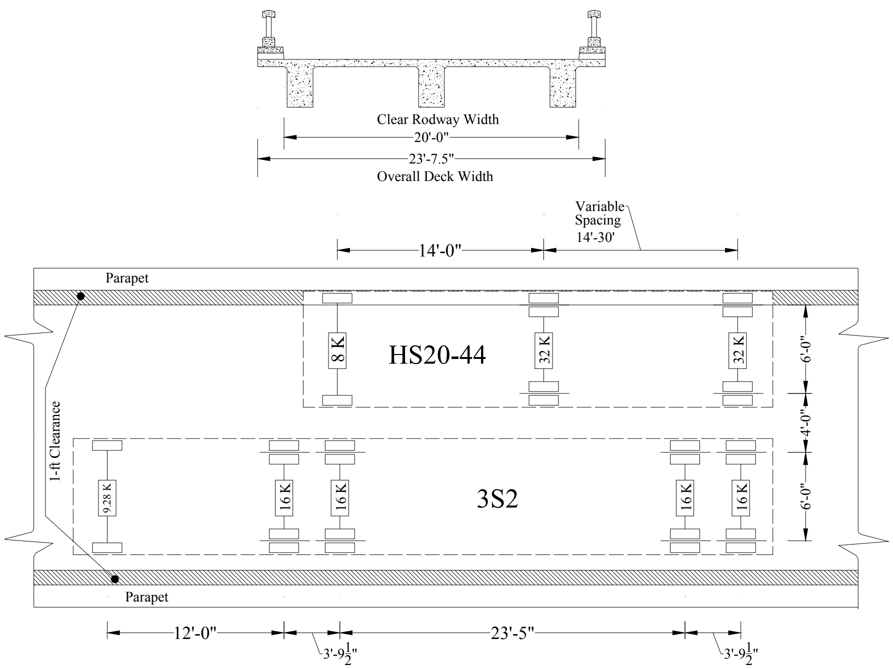


Figure 4 – Truck Load and Truck Lanes

To be noted is that the centerline of the double-wheel of the rear axle shown in Figure 4 is located not less than *1.00 ft* apart from the curb as specified in AASHTO for slab design.

Three loading conditions are required to be checked as laid out in Figure 5.

The HS20-44 design truck load (Figure 5*a*) has a front axle load of *8.0 kip*, second axle load, located *14.0 ft* behind the drive axle, of *32.0 kip*, and rear axle load also of *32.0 kip*. The rear axle load is positioned at a variable distance, ranging between *14.0* and *30.0 ft*. Given the specific bridge geometry, the worst loading scenario is obtained for the minimum spacing of *14.0 ft* between the two rear axles.

The 3S2 design truck load has five axles; the front axle of 9.2*8 kip*, the second double axle, located *12.0 ft* behind the drive axle, of *16.0 kip*, and the rear double axle also of *16.0 kip*, as shown in Figure 5*b)*. Distances between axles are given in the figure.

The load lane condition consists of a load of 640 lbs per linear foot, uniformly distributed in the longitudinal direction with a single concentrated load so placed on the span as to produce maximum stress. The concentrated load and uniform load is considered to be uniformly distributed over a *10'-0"* width on a line normal to the centerline of the lane. The intensity of the concentrated load is represented in Figure 5*c)* for both bending moment and shear force calculations. This load shall be placed in such positions within the load lane as to produce the maximum stress in the member.

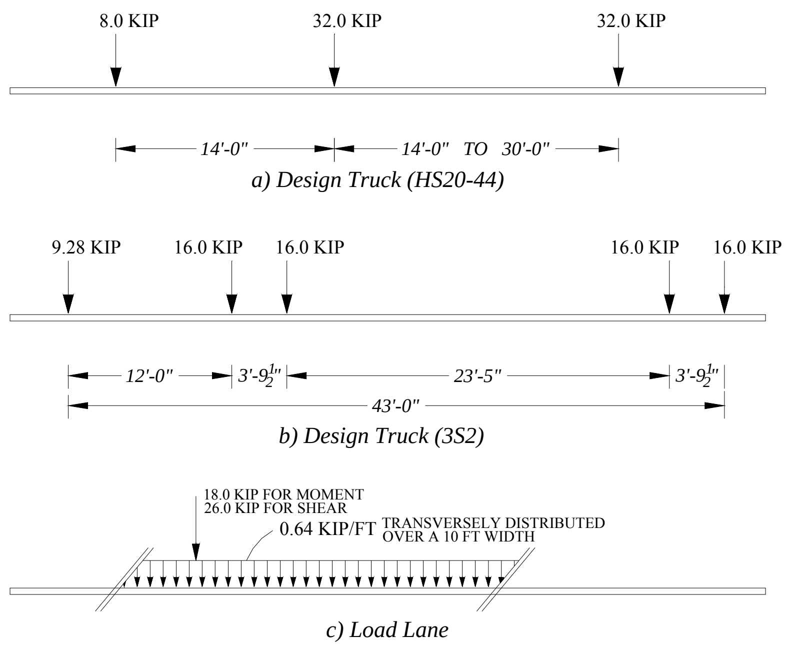


Figure 5 – Loading Conditions

B.3 Slab Analysis

The deck slab is considered to be a one-way slab system due to its large aspect ratio (panel length divided by the panel width). The width of the slab strip to be used in the analysis and design is provided by AASHTO[3] (Table 4.6.2.1.3-1) and, for cast-in-place concrete, may be written as follows for positive and negative moment regions, respectively:

$$\begin{aligned} b^+ &= 26.0 + 6.6S \quad in. \\ b^- &= 48.0 + 3.0S \quad in. \end{aligned} \tag{3}$$

where S represents the center-to-center spacing of the girders *(ft)*.

A generic slab-girder system displaces as shown in Figure 6*a*. This displacement can be seen as the superposition of the displacement associated with local effects represented in Figure 6*b* and the global effect due to the vertical displacement of the girders. Since the local effect is usually significantly greater than the global effect, the latter will be neglected, and the strip analyzed using classical beam theory, assuming that the girders provide rigid support.

The analysis of the slab is carried out on a structure similar to that shown in Figure 6*b)*.

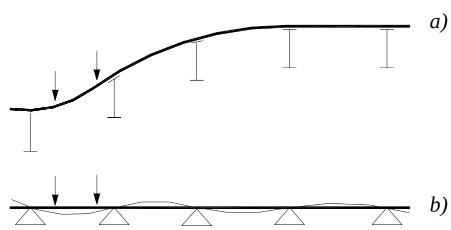


Figure 6 – Slab Deck Deflection due to External Loads

B.3.1 Results of the Analysis

In the following, only the results will be presented. A detailed protocol for the analysis is shown in APPENDIX A and APPENDIX B.

The four loading conditions being considered are shown in Figure 7. The first two loading conditions, *I)* and *II)*, are related to the design truck load. Loading conditions *III)* and *IV)* of Figure 7 refer to the load lane.

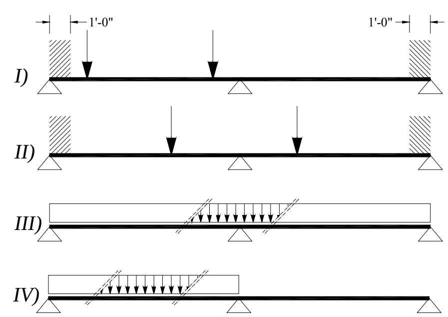


Figure 7 – Loading Conditions for Slab Analysis

Table 1 summarizes the results in terms of ultimate (factored) bending moments and shear forces for the case of wheel loads corresponding to the HS20-44 truck load, which represents the most demanding loading condition. Results of Table 1 do take into account of the moment and shear due to the asphalt layer *(6 in.)* and the self-weight of the deck. The values are adopted for design.

Table 1 – Slab Ultimate Bending Moments and Shear Forces per Unit Strip

Span	Span Length (in)	Loading Condition	Number of Design Lanes Considered	Positive Moment[a)] (k-ft/ft)	Negative Moment[b)] (k-ft/ft)	Shear[c)]	
						V_u (kip/ft)	M_u[d)] (k-ft/ft)
All	108	*I)*	1	12.4	6.3	8.9	3.5
		II)	1	12.5	6.9	7.9	0.8
		III)	2	2.7	1.8	1.9	-
		IV)	1	3.2	1.5	1.8	-

a) Computed close to mid-span depending on load location (See Figure 8)
b) Computed at a cross-section flush with the girder considering 2" chamfer
c) Computed at a cross-section in compliance with AASHTO[3] Section 5.13.3.6.1
d) Computed at the same cross-section where V_u has been evaluated

Figure 8 shows the bending moment diagrams due to the live load only as the design truck load moves transversally on the bridge deck for both loading condition *I)* and *II)*, respectively. These moments were divided by the strip widths shown in Eq. (3) to obtain the values of unit moment summarized in Table 1. Values of Table 1 do take into account the moment due to the dead load as well.

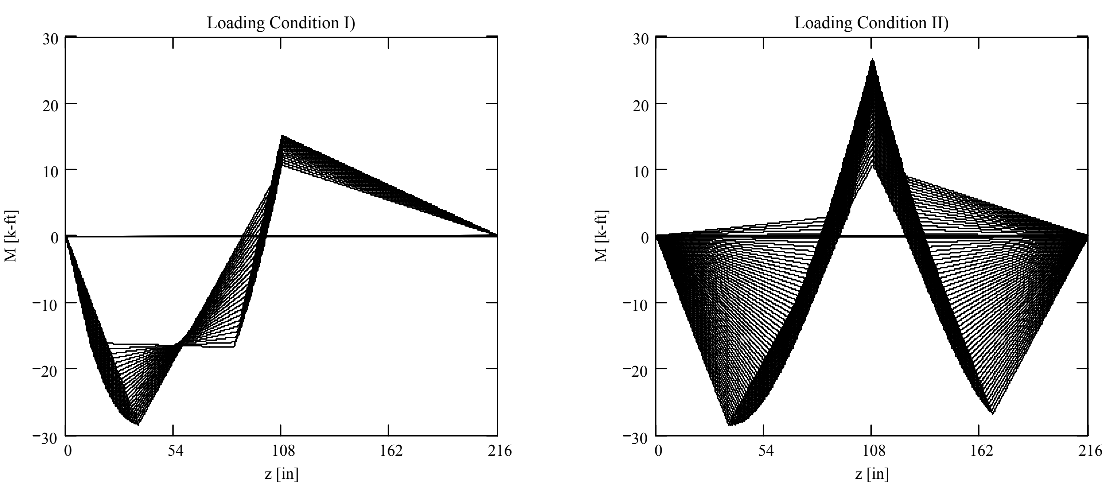


Figure 8 – Live Load Bending Moment Diagram Envelopes per Design Width

B.4 Transversal Load Distribution to Girders

B.4.1 Model for Computing Distribution

According to the truck load arrangement of Figure 4, the transversal load distribution can be found by analyzing the structures in Figure 9, where a generic axle of unit weight has been assumed. Because of geometrical restraints, the truck can not drive on the overhang; hence, the small cantilever portion of the deck has been neglected.

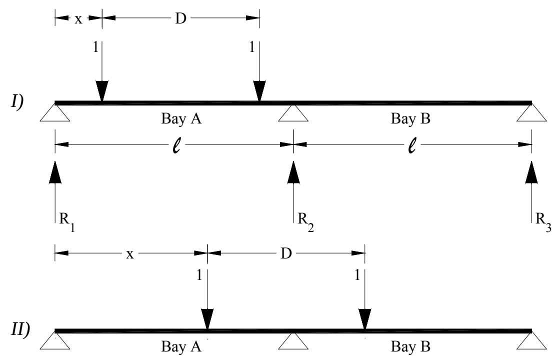


Figure 9 – Transversal Load Distribution: Design Truck Analysis

By increasing the value of *x* represented in Figure 9, the design truck load moves from the left to the right portion of the bridge deck. As this movement is allowed, two possible different loading configurations can be recognized as shown in Figure 9*I)* and *II)*.

The difference between these configurations is related to the number of wheels per bay, as summarized in Table 2. Any other loading condition can be represented by reference to one of the two aforementioned conditions. Table 2 summarizes the values obtained from Figure 9 for the bridge under examination.

Table 2 – Loading Conditions and Bridge Dimensions

Span	Loading Condition	Reference	Bay A	Bay B	x (in)	ℓ (in)	D (in)	d (in)
All	*I)*	Figure 9-*I)*	2 wheels	0 wheel	$12 \le x \le 36$	108	72	48
	II)	Figure 9-*II)*	1 wheel	1 wheel	$36 \le x \le 96$			

A complete analysis of the conditions represented in Figure 9 is carried out according to the protocol of APPENDIX A. In the following, only the results of this analysis are presented.

B.4.2 Results of the Analysis

Figure 10 shows the load lane conditions when two and one design lane are considered. The calculations related to this analysis are summarized in APPENDIX B.

Figure 11 shows each reaction $R_1 \ldots R_3$ of Figure 9 (which represents the load carried by each girder) as a function of x.

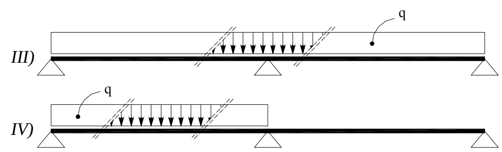


Figure 10 – Transversal Load Distribution: Load Lane Analysis

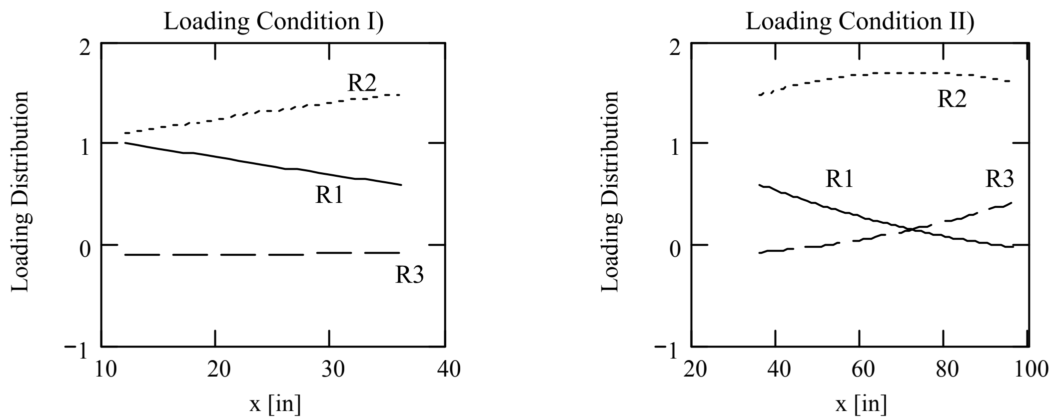


Figure 11 – Reactions as a Function of x

Table 3 summarizes the findings of the distribution of the load to the girders. The k_L coefficient represents the multiplier of the load to be used in the girder analysis.

Table 3 – Vertical Reactions; k_L Coefficient

Coefficient	Span	Loading Condition	Exterior Girders $R_1=R_3$	Interior Girder R_2
k_L	All	*I)*	1.007*a)*	1.481
		II)	0.593	1.704*a)*
		III)	0.375	1.25*b)*
		IV)	0.438*b)*	0.625

a) Design values to be used for the design truck analysis of the girders
b) Design values to be used for the load lane analysis of the girders

B.5 Girders Analysis

B.5.1 Model for Computing Internal Forces and End Reactions

The analysis is conducted for the three loading conditions recognized in Figure 5 namely: *1)* HS20-44 truck load, *2)* 3S2 truck load, and 3*)* load lane.

B.5.1.1 Load Lane Analysis

In the last loading condition of Figure 5*c)*, a uniform load of *0.64 kip/ft* is distributed over the entire length of the girder. Transversely, it is assumed to be uniformly distributed over a *10 ft* width. Hence, the portion of the uniformly distributed load, q, carried by each of the three girders can be expressed as follows:

$$q = (0.64)k_L \tag{4}$$

where k_L represents the fraction of the total load carried by each individual girder. The value of k_L for interior and exterior girders and for central and lateral span is reported in Table 3.

B.5.1.2 HS20-44 and 3S2 Analysis

Figure 12 shows a generic girder with a generic truck load moving on it as the value of x_1 increases from *0* to L.

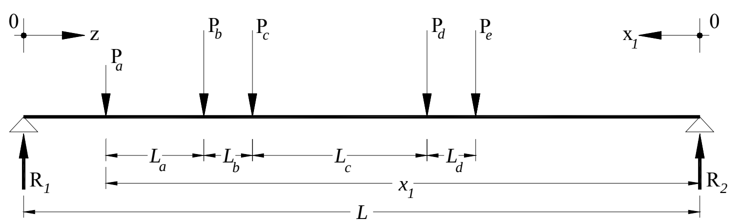


Figure 12 – Design Truck on the Girder

The values of P_i *(i=a,b,c,d,e)* can be expressed as follows:

$$P_i = k_L P_{wi} \tag{5}$$

where P_{wi} is the wheel load defined by AASHTO (*4* and *16 kip*, for an HS20-44 truck load, and *4.64* and *8 kip* for a 3S2 truck load) and k_L represents the fraction of the total load carried by each individual girder. The determination of k_L is presented in APPENDIX A and its value is summarized in Table 3 for both interior and exterior girders and central and lateral span, respectively.

Table 4 summarizes values reported in Figure 12 and Figure 5*c)* for the girders under examination and for the three loading conditions being considered.

As the design truck load moves from the right to the left side of the girder, five different loading conditions are recognized, as shown in Figure 13.

A complete structural analysis protocol for an HS20-44 design truck load is carried out in APPENDIX C.

Table 4 – Parameters for Girder Analysis

Analysis Type	x_1 (ft)	L_a (ft)	L_b (ft)	L_c (ft)	L_d (ft)	L (ft)	P_{wa} (kip)	P_{wb} (kip)	P_{wc} (kip)	P_{wd} (kip)	P_{we} (kip)
HS20-44	Varies	14.0	14.0	0.0	0.0	40	4.0	16.0	16.0	0.0	0.0
3S2	Varies	12.0	3.8	23.4	3.8	40	4.64	8.0	8.0	8.0	8.0
Load Lane [a)]	L/2	0.0	0.0	0.0	0.0	40	18.0 [b)]	0.0	0.0	0.0	0.0
	d[d)]	0.0	0.0	0.0	0.0	40	26.0 [c)]	0.0	0.0	0.0	0.0

Notes: *a)* Related to the concentrated load analysis only; *b)* For bending moment analysis; *c)* For shear force analysis; *d)* Girder effective depth

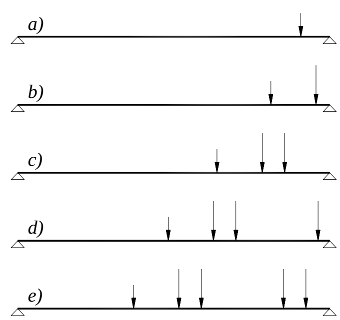
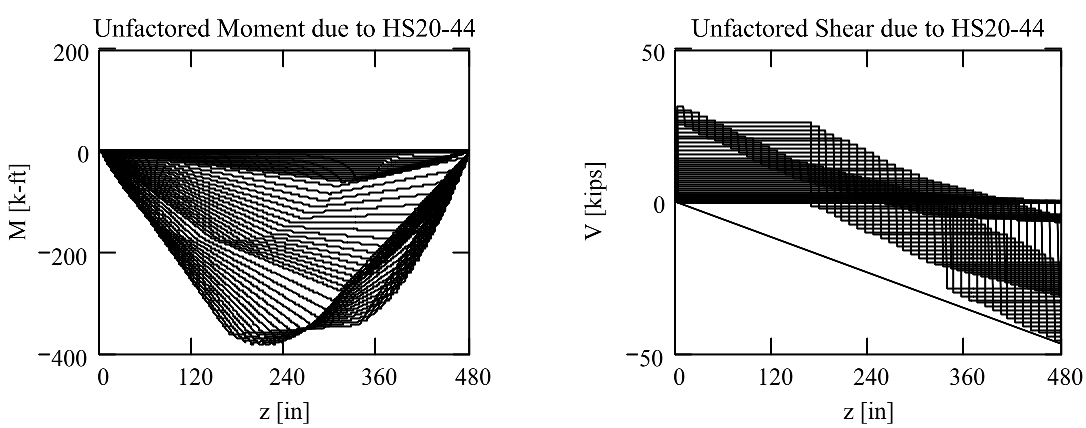


Figure 13 – Design Truck: Possible Loading Conditions

B.5.2 Results of the Analysis

In the following, only the results needed for the design of critical girder cross-sections are presented. Graphical results will be presented only for the case of the design trucks HS20-44 and 3S2. Results related to the load lane analysis will be summarized later in Table 5 and Table 6.

Figure 14 and Figure 15 show the diagrams of both moments and shear forces as the design trucks move on the interior and exterior girders, respectively.



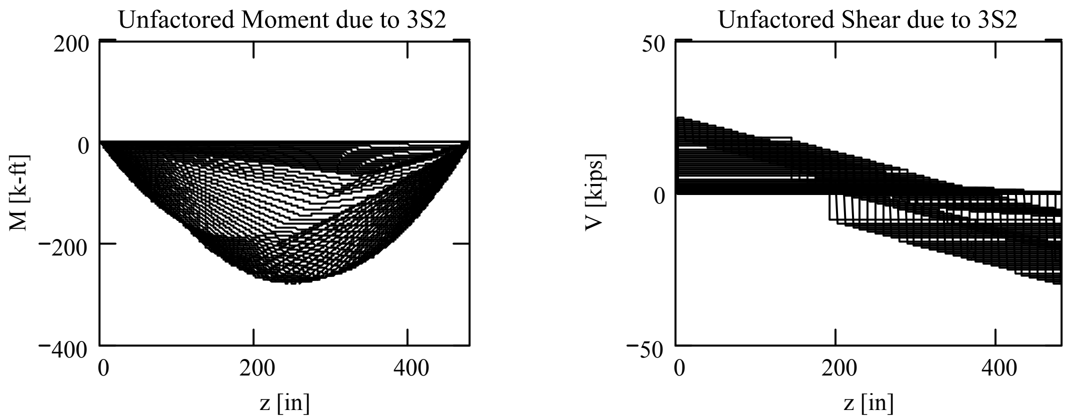


Figure 14 – Live Load Moment and Shear Diagrams for Interior Girders: All Spans

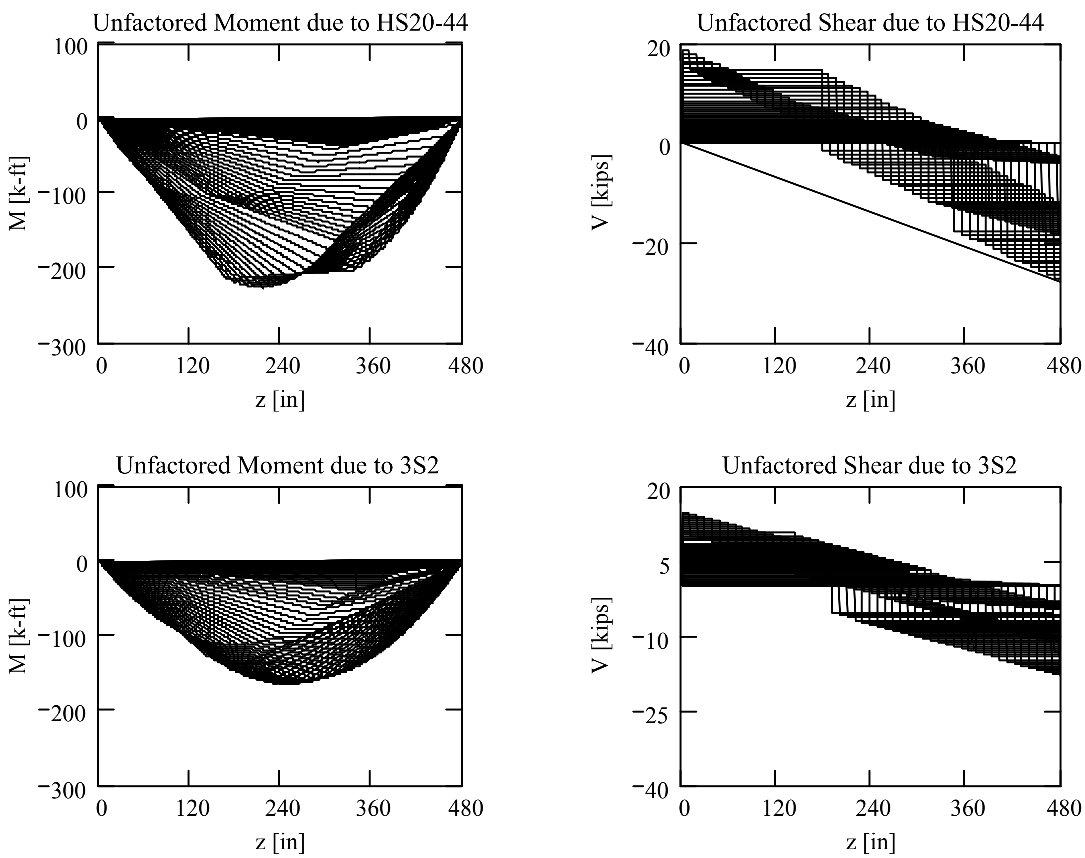


Figure 15 – Live Load Moment and Shear Diagrams for Exterior Girders: All Spans

Bending moment and shear force represented in Figure 14 and Figure 15 refer to both design trucks HS20-44 and 3S2. Ultimate values are then obtained by taking into account the load factors and by adding the moment and shear due to the factored dead load.

B.5.3 Load Combinations and Results

Ultimate bending moments and shear forces calculated at several cross-sections, either at a distance *X* (for moment) or *v* (for shear) from the support (see Figure 16), are summarized in Table 5 and Table 6 for both HS20-44 design truck load and load lane. Loading condition 3S2 is not reported because does not control the design as already shown in Figure 14 and Figure 15. The reported values do take into account both factored dead and live load. The cross-sections indicated in Table 5 and Table 6 (i.e, 1-1, A-A etc.) were shown to be critical locations in a preliminary analysis.

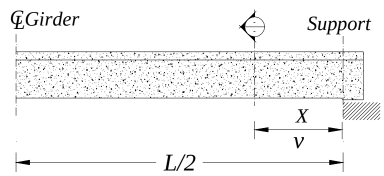


Figure 16 – Identification of Girders Critical Cross-Sections

Table 5 – Breakdown of Moment at Critical Cross-Sections (k-ft)

Span	Girder	Section		Dead Load	Live Load		Factored Load	
		Description	*X* (in.)		HS20-44	Lane	HS20-44	Lane
All	Exterior	Support	0	0	0	0	0	0
		1-1	51	140.7	101.6	59.5	467.5	349.6
		2-2	105	253.2	173.7	117.0	815.6	656.9
		3-3	141	307.4	210.3	152.2	988.6	825.9
		4-4	195	357.4	224.6	200.3	1093.6	1025.5
		Mid-span	240	370.4	226.4	236.0	1115.6	1142.5
	Interior	Support	0	0	0	0	0	0
		1-1	51	120.5	172.0	99.0	634.6	434.0
		2-2	105	216.8	294.0	188.1	1098.9	808.7
		3-3	141	263.2	340.7	238.5	1289.0	1012.2
		4-4	195	306.0	380.1	300.6	1454.1	1239.8
		Mid-span	240	317.2	383.1	340.0	1476.9	1364.6

Table 6 – Breakdown of Shear at Critical Cross-Sections (kip)

Span	Girder	Section		Dead Load	Live Load		Factored Load	
		Description	v (in.)		HS20-44	Lane	HS20-44	Lane
All	Exterior	Support	0	37.0	27.8	29.9	126.0	132.7
		A-A	36	31.5	25.1	6.4	111.2	59.0
		B-B	51	29.2	23.9	6.1	104.9	55.0
		C-C	105	20.8	19.9	4.8	82.8	40.6
		D-D	177	9.7	14.6	3.1	53.5	21.5
		Mid-span	240	0	9.8	1.7	27.4	4.7
	Interior	Support	0	31.7	47.0	40.3	171.8	155.2
		A-A	36	27.0	42.2	15.3	152.9	78.2
		B-B	51	25.0	40.5	14.3	145.0	72.8
		C-C	105	17.8	33.6	10.7	116.6	53.3
		D-D	177	8.3	24.7	5.9	79.5	27.4
		Mid-span	240	0	16.6	1.7	46.1	4.7

B.6 Bent Analysis

B.6.1 Model for Computing Vertical Reactions

Bent analysis is carried out considering the structure shown in Figure 17. Vertical loads P_1, P and P_2 are calculated as follows:

$$
\begin{aligned}
P_1 &= (1.3)R_{D,Ext} + (1.3)(1.67)(1.29)R_{L,Ext} \\
P &= (1.3)R_{D,Int} + (1.3)(1.67)(1.29)R_{L,Int} \\
P_2 &= (1.3)R_{D,Ext} + (1.3)(1.67)(1.29)R_{L,Ext}
\end{aligned}
\qquad (6)
$$

where R_D and R_L are the vertical reactions due to the dead and live load of girders and deck, respectively, and subscripts *Ext* and *Int* refer to exterior and interior girders.

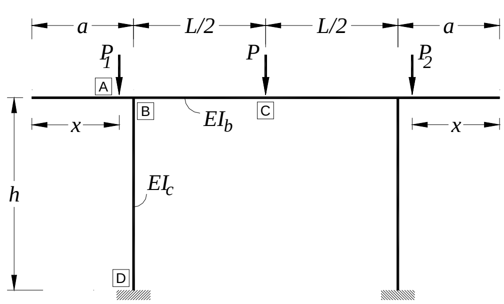


Figure 17 – Bent Frame

Table 7 summarizes the geometrical properties of the frame of Figure 17.

Table 7 – Frame Geometrical Properties

L (in)	*a* (in)	*x* (in)	*h* (in)
180.0	42.0	20.0	215.0

B.6.1.1 Dead Load Analysis

The reactions due to the dead load can be calculated as follows (see Figure 18):

$$R = \frac{\omega_d L}{2} \qquad \omega_d = h^* W(\gamma_a + \gamma_c) + 3bh\gamma_c \tag{7}$$

where L represents the length of the girder, $h^*=h_a=h_s$ *(6 in.)*, and γ_a and γ_c are the weight per cubic foot of asphalt and concrete, *108* and *150 pcf* respectively.

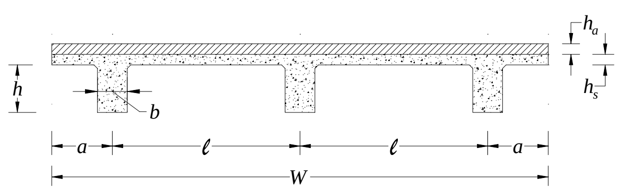


Figure 18 – Determination of the Dead Load of the Superstructure

By neglecting the presence of both overhangs and parapets (conservative assumption), the sharing of the dead load between girders is reported in Figure 19.

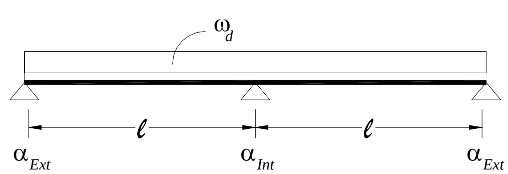


Figure 19 – Load Sharing Between Girders

Table 8 summarizes the findings of the application of Eq. (7).

Table 8 – Application of Eq. (7)

Span	*W* (ft)	*L* (ft)	ω_d (k/ft)	*R* (kip)	$R_{tot}=2R$ (kip)
All	23.75	40.0	4.339	86.8	173.6

The vertical reactions due to the dead load to be considered for the bent analysis are summarized in Table 9. Their values do consider the sharing distribution of Figure 19 according to the following equation:

$$R_D = \alpha R_{tot} \tag{8}$$

Table 9 – Reactions due to Dead Load

Girder Type	Sharing of Load α	Vertical Reaction R_D (kip)
Exterior	0.1875	23.6
Interior	0.625	108.5

B.6.1.2 Live Load Analysis: HS20-44

The loading condition imposed on the bent from the superstructure being considered in this analysis is shown in Figure 20. The load of the figure is related to the wheel load of an HS20-44.

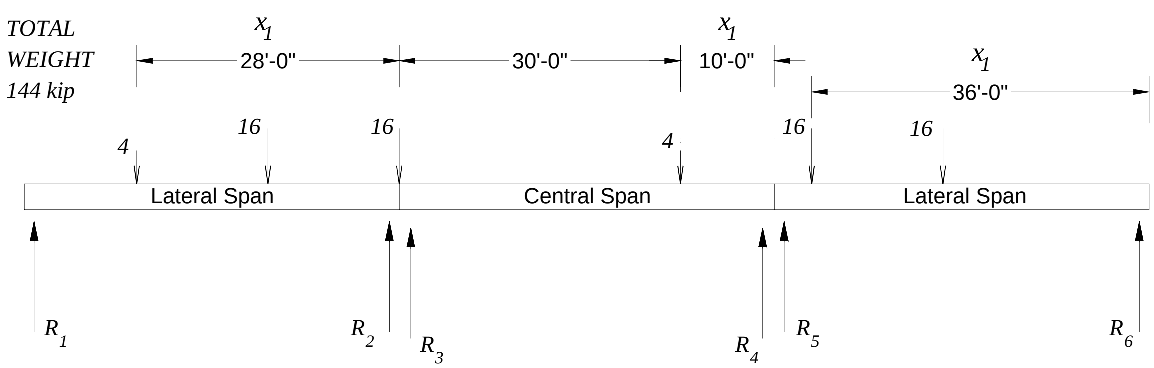


Figure 20 – Bent Loading Condition (Wheel Loads Shown)

Table 10 and Table 11 summarize the results obtained when the loading arrangement on the deck is that of Figure 21. This loading condition corresponds to a truck that moves transversally on the bridge deck.

Table 10 – Vertical Reactions of the Girders due to an HS20-44

Loading Condition (Figure 21)	Span	k_L	Girder Type (Figure 21)	R_1 (kip)	R_2 (kip)	R_3 (kip)	R_4 (kip)	R_5 (kip)	R_6 (kip)
Ia)	Lateral	1.007	Exterior 1	8.5	27.8	0	0	23.4	8.9
		1.097	Interior	9.2	30.3	0	0	25.5	9.7
		-0.105	Exterior 2	-0.9	-2.9	0	0	-2.4	-0.9
	Central	1.007	Exterior 1	0	0	1.0	3.0	0	0
		1.097	Interior	0	0	1.1	3.3	0	0
		-0.105	Exterior 2	0	0	-0.1	-0.3	0	0
Ib)=IIa)	Lateral	0.593	Exterior 1	5.0	16.4	0	0	13.8	5.2
		1.481	Interior	12.4	40.9	0	0	34.4	13.0
		-0.074	Exterior 2	-0.6	-2.0	0	0	-1.7	-0.7
	Central	0.593	Exterior 1	0	0	0.6	1.8	0	0
		1.481	Interior	0	0	1.5	4.4	0	0
		-0.074	Exterior 2	0	0	-0.1	-0.2	0	0
IIb)	Lateral	0.149	Exterior 1	2.3	4.1	0	0	3.5	1.3
		1.704	Interior	14.3	47.0	0	0	39.5	15.0
		0.149	Exterior 2	2.3	4.1	0	0	3.5	1.3
	Central	0.149	Exterior 1	0	0	0.1	0.4	0	0
		1.704	Interior	0	0	1.7	5.1	0	0
		0.149	Exterior 2	0	0	0.1	0.4	0	0

Table 11 – Vertical Loads Acting on the Bent due to an HS20-44

<table>
<tr><th rowspan="2">Loading Condition</th><th colspan="3">R_2+R_3 (kip)</th><th colspan="3">R_4+R_5 (kip)</th><th rowspan="2">R_{tot} (kip)</th></tr>
<tr><th>Exterior 1</th><th>Interior</th><th>Exterior 2</th><th>Exterior 1</th><th>Interior</th><th>Exterior 2</th></tr>
<tr><td>Ia)</td><td>28.8</td><td>31.4</td><td>-3.0</td><td>26.4</td><td>28.8</td><td>-2.7</td><td>144.2</td></tr>
<tr><td>Ib)=IIa)</td><td>17.0</td><td>42.4</td><td>-2.1</td><td>15.6</td><td>41.8</td><td>-1.9</td><td>147.1</td></tr>
<tr><td>IIb)</td><td>4.2</td><td>48.7</td><td>4.2</td><td>3.9</td><td>44.6</td><td>3.9</td><td>146.0</td></tr>
</table>

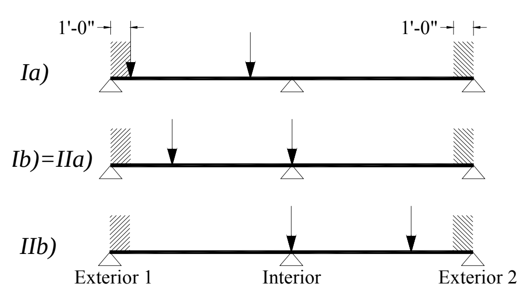


Figure 21 – Transversal Loading Conditions

Table 12 summarizes the values R_L due to the design truck load to be used in Eq. (6).

Table 12 – Reactions due to HS20-44

Loading Condition	Girder Type	Vertical Reaction R_L (kip)
Ia)	Exterior 1	28.8
	Interior	31.4
	Exterior 2	-3.0
Ib)=IIa)	Exterior 1	17.0
	Interior	42.4
	Exterior 2	-2.1
IIb)	Exterior 1	4.2
	Interior	48.7
	Exterior 2	4.2

B.6.1.3 Live Load Analysis: Load Lane

The analysis related to the load lane takes into account for both the uniformly distributed load and the concentrated load. For the latter case, only the load of *P=26 kip* of Figure 5*c)* will be considered in the analysis since it yields to the worst loading condition scenario. The analysis is performed on the structures shown in Figure 22 considering two design lanes.

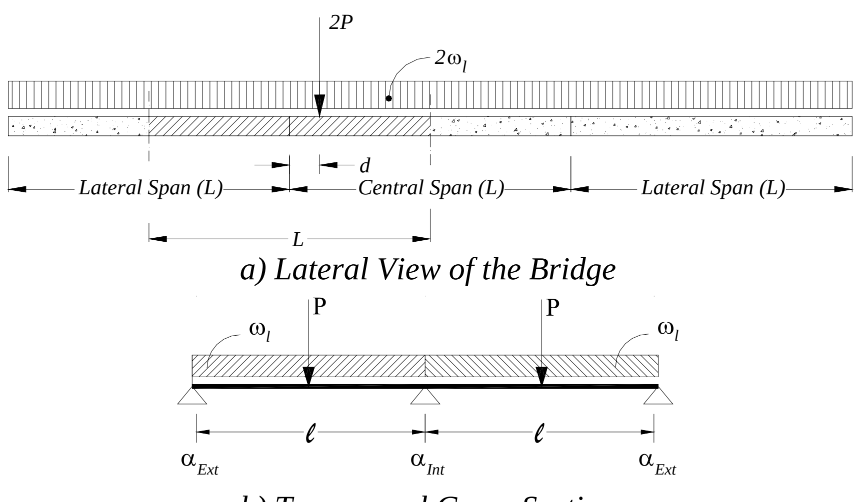


Figure 22 – Loading Condition for the Load Lane Analysis

The vertical reactions R_L can be expressed as follows:

$$R_L = \begin{cases} 2\alpha_u \omega_L L & \text{uniform load analysis} \\ 2\alpha_c P \dfrac{L-d}{L} & \text{concentrated load analysis} \end{cases} \tag{9}$$

where α_u and α_c are the sharing of the load between girders (see Table 13), ω_L*=0.64 kip/ft*, *L=40 ft*, and *d=27.5 in*. The results of the application of Eq. (9) are highlighted in Table 13. Table 14 summarizes the load to be used in Eq. (6) when using the load lane analysis.

Table 13 – Application of Eq. (9)

Load Type	Girder Type	Sharing of Load α_u and α_c	Vertical Reaction R_L (kip)
Uniform	Exterior	0.1875	9.6
	Interior	0.625	32.0
Concentrated	Exterior	0.15625	7.7
	Interior	0.6875	33.7

Table 14 – Reactions due to Load Lane

Girder Type	Vertical Reaction R_L (kip)
Exterior	9.6+7.7=17.3
Interior	32.0+33.7=65.7

B.6.2 Vertical Reactions: Results

The application of Eq. (6) gives the results summarized in Table 15 for the two loading conditions namely: 1) HS20-44; and 2) load lane.

Table 15 – Load Transferred to the Bent

Analysis	Loading Condition	P_1 (kip)	P (kip)	P_2 (kip)
HS20-44	*Ia)*	111.3	229.0	22.3
	Ib)=IIa)	78.3	259.8	24.8
	IIb)	42.4	277.4	42.4
Load Lane	2 Lanes	79.1	325.0	79.1

B.6.3 Load Combination and Results

Table 16 summarizes ultimate moments and shear forces calculated at five cross-sections where maximum values are reached. Bending moment and shear force diagrams for the loading condition of Table 15 are shown in Figure 23. A detailed calculation protocol is provided in APPENDIX D.

Table 16 – Ultimate Moment and Shear

Analysis	Loading Condition	Section	M_u (k-ft)	V_u (kip)
HS20-44	*Ia)*	B-A	204.1	111.3
		B-C	309.8	123.5
		C-B	616.2	123.5
		B-D	134.5	13.5
		D-B	67.3	13.5
	Ib)=IIa)	B-A	143.5	78.3
		B-C	288.5	135.3
		C-B	726.1	135.3
		B-D	162.3	16.2
		D-B	81.2	16.2
	IIb)	B-A	77.7	42.4
		B-C	250.9	138.7
		C-B	789.4	138.7
		B-D	173.1	17.3
		D-B	86.6	17.3
Load Lane	2 Lanes	B-A	145.0	79.1
		B-C	326.7	162.5
		C-B	892.0	162.5
		B-D	181.7	18.2
		D-B	90.9	18.2

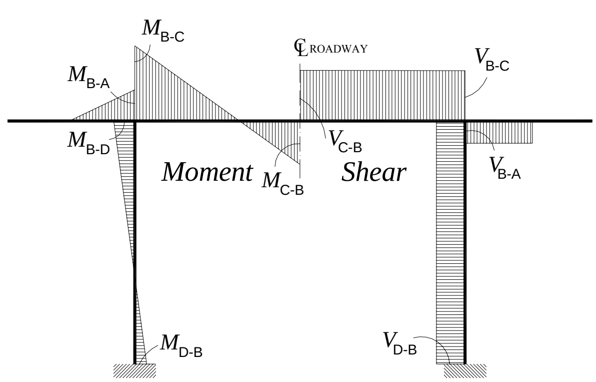


Figure 23 – Bending Moment and Shear Force Diagram

For non-symmetric loading condition such as *Ia)* and *Ib)=IIa)*, values of maximum moment and shear have been considered. They do not necessarily occur at the indicated cross-sections and could be located on the portion of the moment and shear diagram not represented in Figure 23.

In Table 16 and Figure 23 only the results corresponding to the live load are presented because the contribution of the self weight of the member can be neglected.

C. DESIGN

C.1 Assumptions

Strengthening design is carried out according to the principles of ACI 440.2R-02 (ACI 440 in the following). The properties of concrete, steel, and FRP and SRP laminates used in the design are summarized in Table 17. The reported FRP and SRP properties are guaranteed values. The FRP and SRP systems used in the design of this bridge are highlighted in Table 17.

The ϕ factors used to convert nominal values to design capacities are obtained as specified in AASHTO for the as-built and from ACI 440 for the strengthened members.

Table 17 – Material Properties

Concrete f'_c (psi)	Steel f_y (ksi)	System Type				System Tensile Strength f^*_{fu} (ksi)	System Modulus of Elasticity E_f (ksi)	System Size or Thickness t_f (in)
		NSM System	Manual Lay-up	SRP 3X2	SRP 3SX			
6,845	40	Type-1*b*	-	-	-	300	19,000	4/8 bar
		-	Type-2	-	-	550	33,000	0.0065
		-	-	Type-4	-	460	30,000	0.0173
		-	-	-	Type-5	345	30,000	0.0104

Material properties of the composite reinforcement reported by manufacturers, such as the ultimate tensile strength, typically do not consider long-term exposure to environmental conditions, and should be considered as initial properties. Composite properties to be used in all design equations are given as follows (ACI 440):

$$f_{fu} = C_E f^*_{fu}$$
$$\varepsilon_{fu} = C_E \varepsilon^*_{fu} \tag{10}$$

where f_{fu} and ε_{fu} are the FRP or SRP design tensile strength and ultimate strain considering the environmental reduction factor (C_E) as given in Table 8.1 (ACI 440), and f^*_{fu} and ε^*_{fu} represent the FRP or SRP guaranteed tensile strength and ultimate strain as reported by the manufacturer (see Table 17). An environmental reduction factor for SRP is not given in ACI 440 as the document does not directly address designs using SRP. The values used for design of SRP are taken as the values given in ACI 440 for carbon fiber.

The SRP material chosen for this project has a brass coating which inhibits corrosion. The FRP and SRP design modulus of elasticity is the average value as reported by the manufacturer. FRP properties in the case of the NSM system relate to the gross section whereas in the case of manual lay-up relate to net fiber area.

C.2 Slab Design

C.2.1 Assumptions

Slab geometrical properties and internal steel flexural reinforcement are summarized in Table 18.

Table 18 – Slab Geometrical Properties and Internal Steel Reinforcement

Span	Slab Thickness h_s (in)	Slab Width b (in)	Tensile Steel Area A_s (in^2/ft)	Effective Depth d (in)	Compression Steel Area A'_s (in^2/ft)	Effective Depth d' (in)
Both Spans	6	12	#4@5"=0.48	4.75	#4@5"=0.48	1.75

C.2.2 Positive Moment Strengthening (FRP)

The strengthening recommendations summarized in Table 19 are suggested for the case of mid-span location (maximum positive moment) for both central and lateral span.

Table 19 – Slab Positive Moment Capacity

FRP Type	Span	Strengthening Scheme	Failure Mode[a)]	ϕM_n (k-ft/ft)	M_u (k-ft/ft)
Type-2	All	No FRP	CC	7.7	12.5
		1 Ply 6" wide @14" ocs	TC	12.6	

a) CC=Concrete Crushing, TC=Tension Controlled.

When adding FRP, the failure mode is usually governed by FRP rupture because of its limited ultimate strain at failure as compared to that of steel. This also represents an optimal use of an expensive material. Only when the amount of applied FRP becomes larger, the failure mode changes from tension controlled (FRP rupture) to concrete crushing.

Slab flexural strengthening of positive moment region is shown in Figure 24 and Figure 25, respectively.

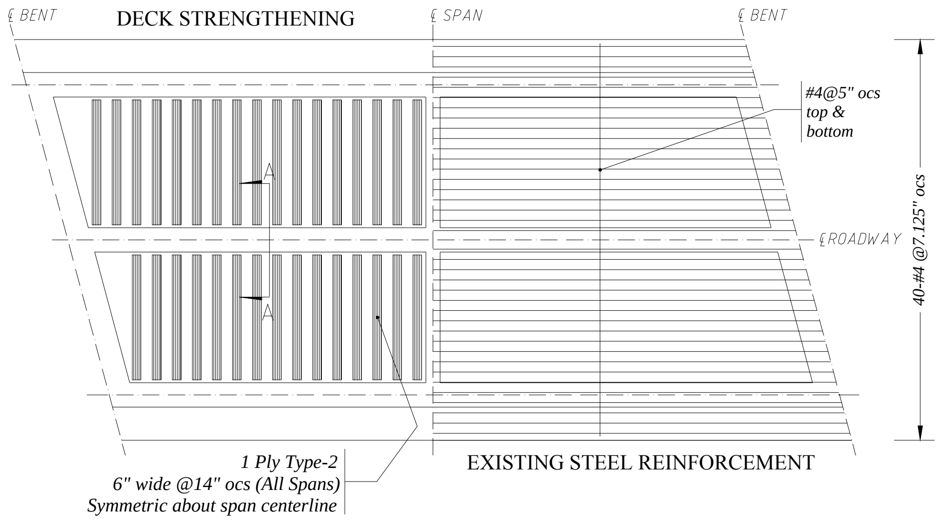


Figure 24 – Plan View of Slab Strengthening (FRP)

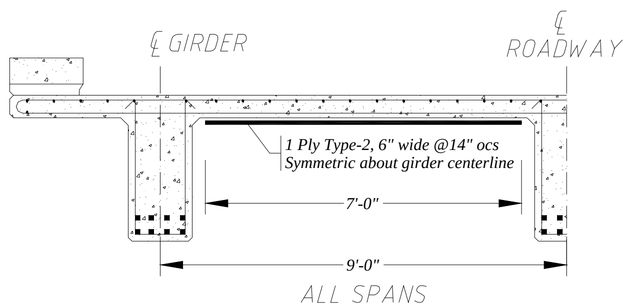


Figure 25 – Slab Cross-Section (FRP)

C.2.3 Positive Moment Strengthening (SRP)

The strengthening recommendations summarized in Table 20 are suggested for the case of mid-span location (maximum positive moment) for both central and lateral span.

Table 20 – Slab Positive Moment Capacity

FRP Type	Span	Strengthening Scheme	Failure Mode[a)]	ϕM_n (k-ft/ft)	M_u (k-ft/ft)
Type-4	All	No SRP	CC	7.7	12.5
		1 Ply 4” wide @ 20” ocs	TC	12.5	

a) CC=Concrete Crushing, TC=Tension Controlled.

Slab flexural strengthening of positive moment region is shown in Figure 26 and Figure 28.

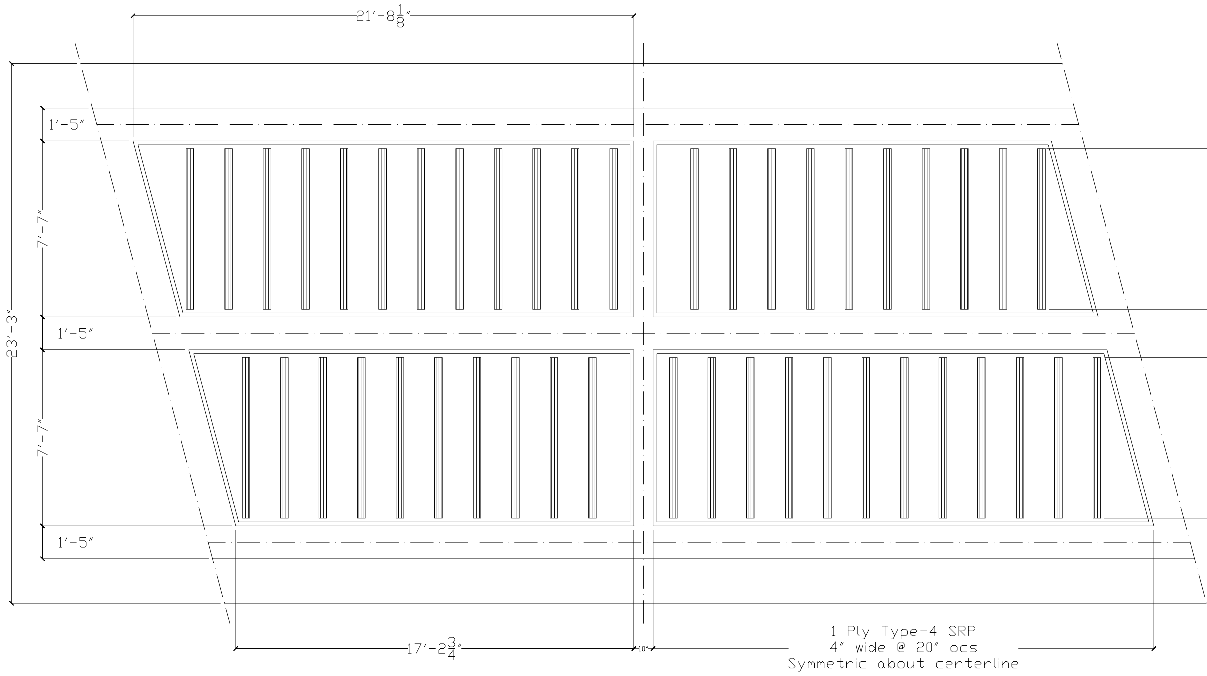


Figure 26 – Plan View of Slab Strengthening (SRP)

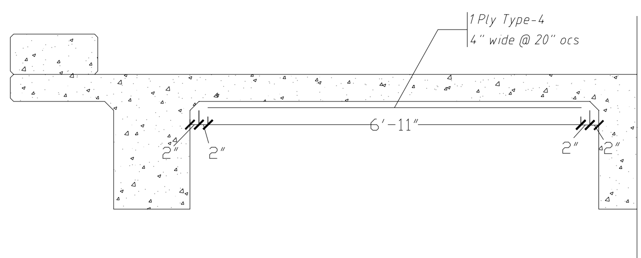


Figure 27 – Slab Cross-Section (SRP)

C.2.4 Negative Moment Check

Strengthening of the negative moment region of the bridge deck is not a viable solution. The as-built negative moment capacity is summarized in Table 21. No flexural strengthening is needed because the as-built capacity is acceptable.

Table 21 – Slab Negative Moment Capacity

Span	Failure	ϕM_n	M_u

	Mode	(k-ft/ft)	(k-ft/ft)
All	CC	6.6	6.9

CC=Concrete Crushing

C.2.5 Shear Check

Shear strengthening of slab-deck systems is not a viable solution. The as-built shear capacity is summarized in Table 22. No shear strengthening will be provided on the slab since the values of the as built shear capacity and demand are sufficiently close.

Table 22 – Slab Shear Capacity

Span	ϕV_n (kip/ft)	V_u (kip/ft)
All	7.7	8.9

The concrete shear capacity of the slab has been calculated using the detailed equation of ACI 318-99[4] (see later Eq. (12)). Ultimate moment at the same cross-section is indicated in Table 1 *(3.5 k-ft/ft)*.

Table 22 reports data for V_u computed considering the resultant load from each set of two tires. Since each tire has a width of *20 in.* (AASHTO Section 3.30), the distributed load would have a total width of *4x20=80 in.* over a span of *108 in*. If one were to consider this load uniformly distributed the equivalent load per linear foot would be *q=0.56 kip/ft*, which would correspond to an ultimate shear computed at the same location of Table 22 of V_u*=7.4 kip/ft*. This value represents a lower bound while the one in Table 22 is the upper bound.

C.3 Girders Design

C.3.1 Assumptions

Girder geometrical properties are reported in Figure 28*a)* and Table 23. Figure 28*b)* and *c)*, Table 24 and

Table 25 summarize internal flexural and shear reinforcement at different cross-sections where there is a change in the lay-out of the reinforcement.

The expression for the flange width, b_{eff}, is given by the following equations, according to AASHTO Section 8.10.1 for interior and exterior girders, respectively:

$$
\begin{aligned}
b_{eff}^{Int} &= \min\left(\frac{L}{4}, 12h_s + b, \ell\right) \\
b_{eff}^{Ext} &= b + \min\left(\frac{L}{12}, 6h_s, \frac{\ell - b}{2}\right)
\end{aligned}
\tag{11}
$$

where L is the girder length, h_s and b are defined in Table 23, and ℓ represents the center-to-center girder spacing.

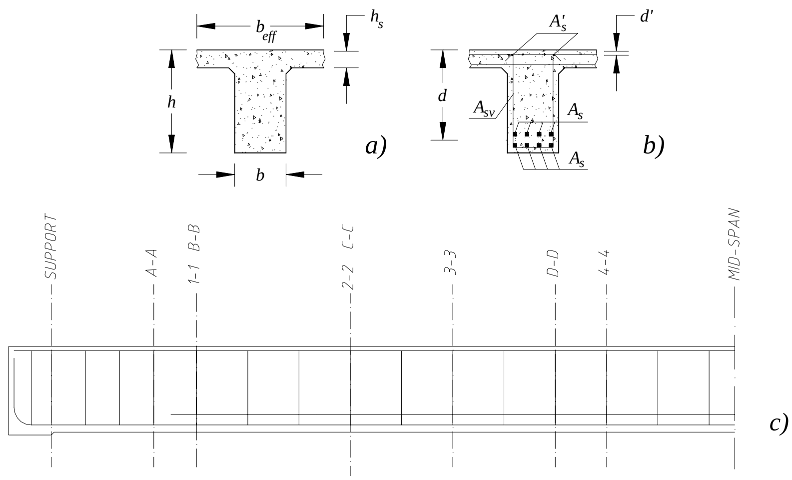


Figure 28 – Girder Dimensions and Internal Reinforcement

Table 23 – Geometrical Properties

Span	Girder Type	Overall Height h, (in)	Width of the Web b, (in)	Width of the Flange b_{eff}, (in)	Slab Thickness h_s, (in)
All	Interior	30	17	89	6
	Exterior	30	17	53	6

Table 24 – Flexural Internal Steel Reinforcement

Span	Girder Type	Section (Figure 28)	Tensile Steel Area A_s (in^2)	Effective Depth d (in)	Compression Steel Area A'_s (in^2)	Effective Depth d' (in)
All	All	Support	6.24	27.5	3.56 Int/2.12 Ext	1.75
		1-1	9.36	26.25	3.56 Int/2.12 Ext	1.75
		2-2 to Mid-span	12.48	25.625	3.56 Int/2.12 Ext	1.75

Table 25 – Shear Internal Steel Reinforcement

Span	Girder Type	Section (see Figure 28)	Stirrup Area A_{vs} (in^2)	Stirrups Spacing s_s (in)
All	All	Support	0.40	12
		A-A	0.40	15
		B-B to Mid-span	0.40	18

C.3.2 Positive Moment Strengthening (FRP)

Table 26 summarizes the achieved flexural capacity at mid-span for interior and exterior girders as a function of the adopted strengthening scheme.

When FRP laminates are used, the bond dependent coefficient, κ_m, defined by Eq. (9-2) of ACI 440, accounts for cover delamination or FRP debonding that could occur if the force in the FRP cannot be sustained by the substrate. When FRP U-Wraps are installed to anchor the external flexural reinforcement, the value of κ_m can be taken up to *0.9* since both cover delamination and FRP debonding are effectively prevented.

Table 26 – Girders Flexural Capacity at Mid-span (FRP)

FRP Type	Span	Girder Type	Description	κ_m (-)	ϕM_n (k-ft)	M_u (k-ft)
Type-2	1-2, 3-4	Interior	No FRP	-	944.2	1476.9
			5 Plies 16” wide + 4 lateral CFRP bars #4	0.9	1476.5	
		Exterior	No FRP	-	928.5	1142.5
			3 Plies 16” wide	0.9	1157.0	

Figure 29 shows the flexural demand and the as built and strengthened capacities of interior and exterior girders, respectively. The demand has been shown for the three loading conditions analyzed.

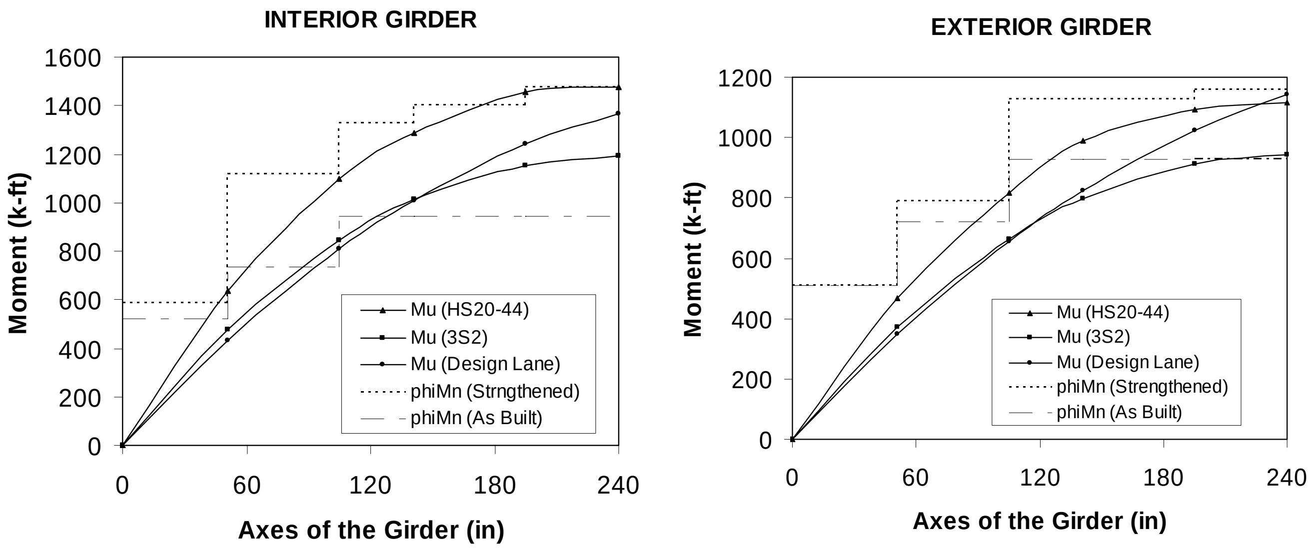


Figure 29 – Flexural Demand and Flexural Capacity (FRP)

A sketch showing the layout of FRP flexural reinforcement is presented in Figure 30. As the ultimate moment decreases towards the supports, the total number of plates reduces.

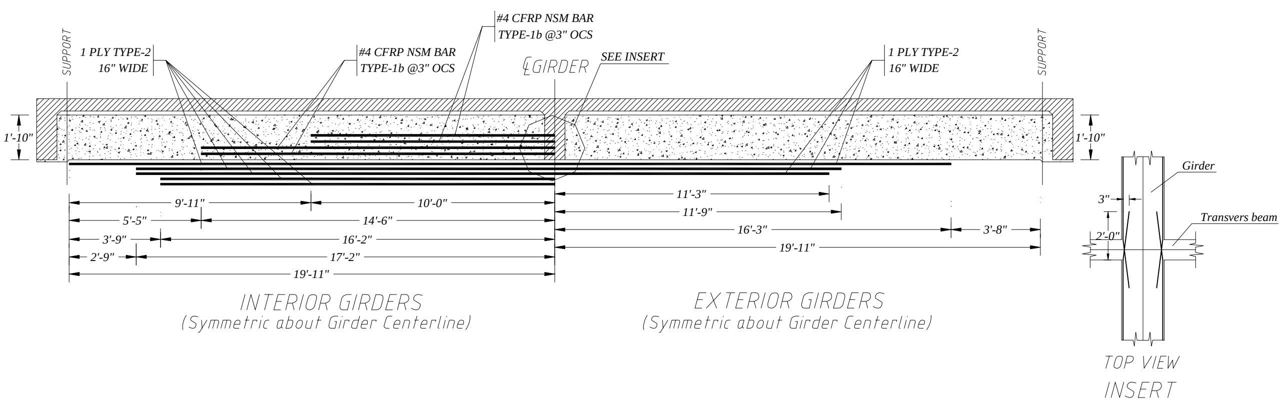


Figure 30 – FRP Flexural Strengthening

C.3.3 Positive Moment Strengthening (SRP)

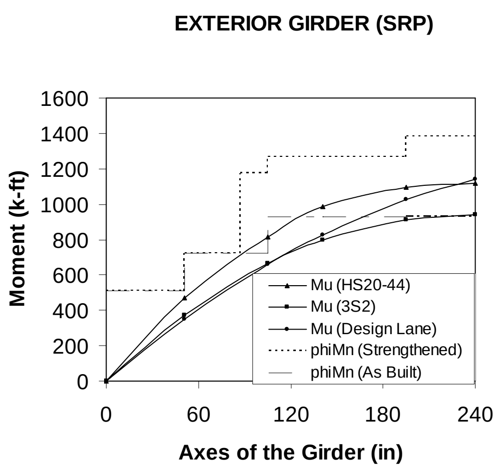
Table 27 summarizes the achieved flexural capacity at mid-span for interior and exterior girders as a function of the adopted strengthening scheme.

When SRP laminates are used, the bond dependent coefficient, κ_m, defined by Eq. (9-2) of ACI 440, accounts for cover delamination or SRP debonding that could occur if the force in the SRP cannot be sustained by the substrate. When SRP U-Wraps are installed to anchor the external flexural reinforcement, the value of κ_m can be taken up to *0.9* since both cover delamination and FRP debonding are effectively prevented.

Table 27 – Girders Flexural Capacity at Mid-span (SRP)

FRP Type	Span	Girder Type	Description	κ_m (-)	ϕM_n (k-ft)	M_u (k-ft)
Type-4	3-4	Interior	No SRP	-	944.2	1476.9
			3 Plies 16" wide	0.9	1452.9	
		Exterior	No SRP	-	928.5	1142.5
			2 Plies 16" wide	0.9	1380.0	

Figure 31 shows the flexural demand and the as built and strengthened capacities of interior and exterior girders, respectively. The demand has been shown for the three loading conditions analyzed.



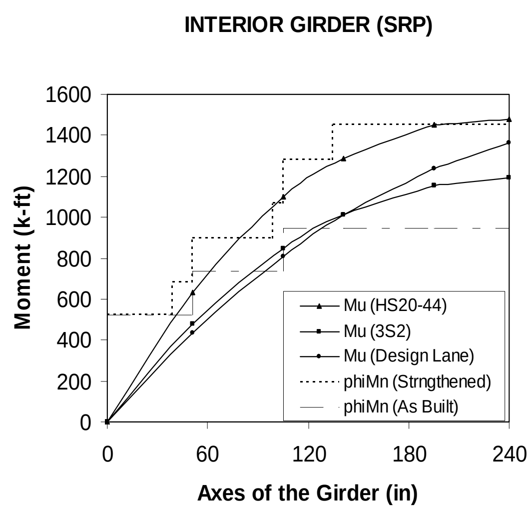


Figure 31 – Flexural Demand and Flexural Capacity (SRP)

A sketch showing the layout of SRP flexural reinforcement is presented in Figure 32. As the ultimate moment decreases towards the supports, the total number of plates reduces.

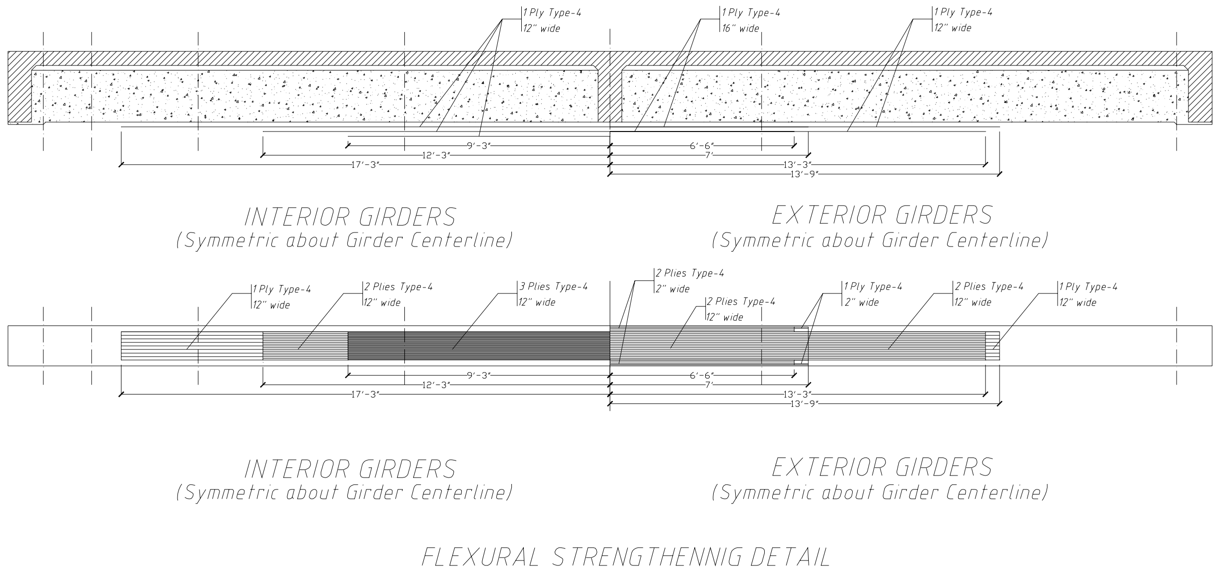


Figure 32 – SRP Flexural Strengthening

C.3.4 Negative Moment Check

All girders are simply supported, and therefore no negative moment exists.

C.3.5 Shear Strengthening (FRP)

The concrete contribution to the shear capacity has been assumed to be based on Eq. (11-5) of ACI 318-99 as follows:

$$V_c = \left(1.9\sqrt{f_c^{'}} + 2500\rho_w \frac{V_u d}{M_u}\right) b_w d \leq 3.5\sqrt{f_c^{'}} b_w d \tag{12}$$

where $\rho_w = A_s / b_w d$, V_u and M_u represent ultimate bending moment and shear force acting at the same cross-section, respectively, and b_w and d are width and effective depth of the girder. The steel contribution to the shear capacity can be expressed as follows:

$$V_s = \frac{A_{vs} f_y d}{s_s} + \frac{A_{vb} f_y (\sin\alpha + \cos\alpha) d}{s_b} \leq 2\frac{A_{vs} f_y d}{s_s} \tag{13}$$

where α represents the slope of the bent bar *(α=45˚)*, and all other symbols are indicated in

Table 25. The limitation expressed by the last term of Eq. (13) is a conservative assumption to take into account for the localized effect exerted by the bent bars.

The FRP contribution to the shear capacity is expressed as follows (ACI 440):

$$V_f = \frac{A_{fv} f_{fe} d_f}{s_f} \tag{14}$$

where A_{fv} is the FRP laminate area, f_{fe} is the effective tensile strength allowable to the FRP reinforcement, d_f is the depth of the FRP reinforcement, and s_f is the FRP spacing.

Table 28 summarizes the achieved shear capacity at support for both interior and exterior girders as a function of the adopted strengthening scheme.

Figure 33 shows the as built and strengthened shear capacities compared to the shear demand for all girders.

Table 28 – Girders Shear Capacity at Support (FRP)

FRP Type	Span	Girder Type	Description	κ_v (-)	ϕV_n (kip)	V_u (kip)
Type-2	1-2, 2-3	Interior	No FRP	-	100.4	171.8
			4 Plies Continuous, U-wrap	0.186	164.2	
		Exterior	No FRP	-	99.5	132.7
			1 Ply Continuous, U-wrap	0.390	123.6	

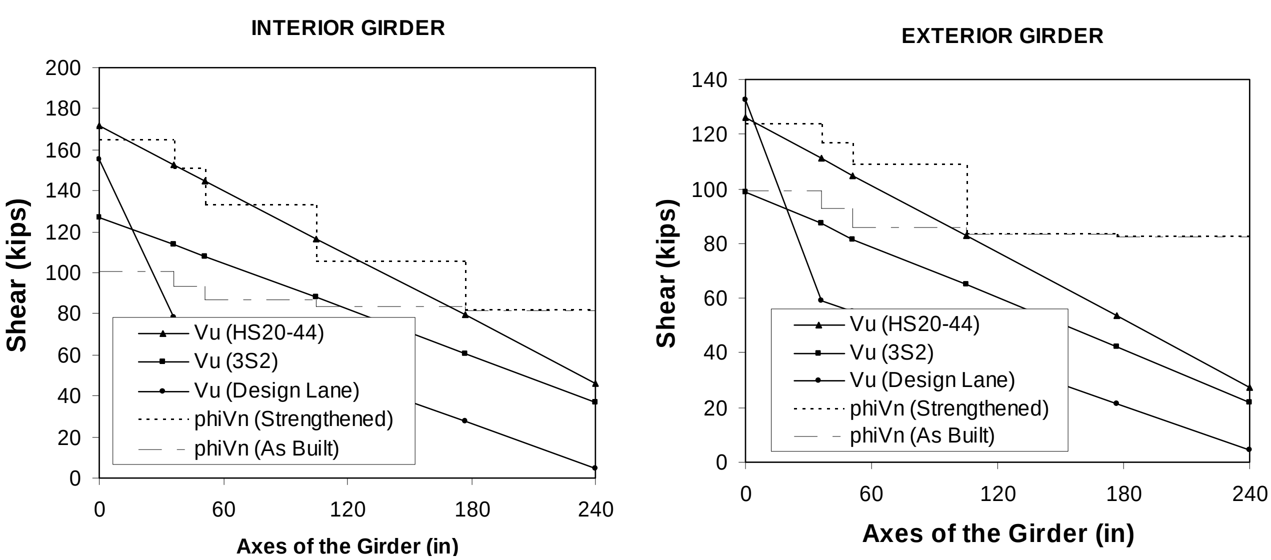


Figure 33 – Shear Demand and Shear Capacity (FRP)

A sketch showing the layout of FRP shear reinforcement is presented in Figure 34. A ½” minimum corner radius needs to be provided when FRP sheet is wrapped around outside corners

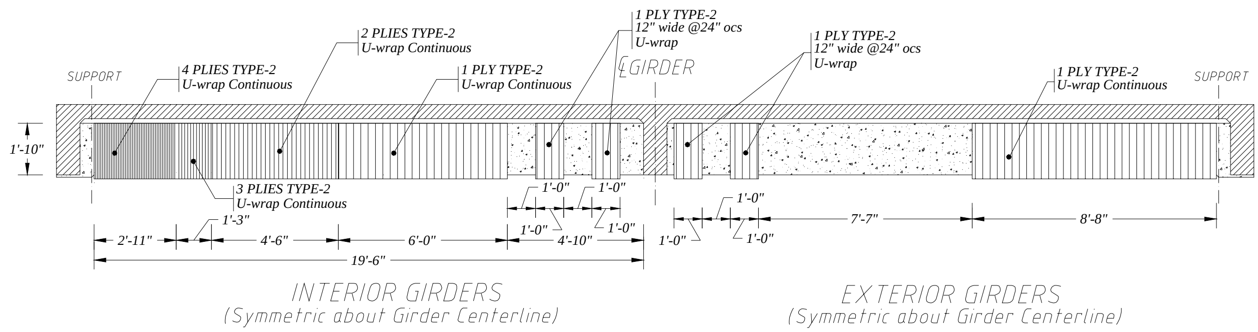


Figure 34 – FRP Shear Strengthening

C.3.6 Shear Strengthening (SRP)

The SRP contribution to the shear capacity is expressed as follows (ACI 440):

$$V_f = \frac{A_{fv} f_{fe} d_f}{s_f} \tag{15}$$

where A_{fv} is the FRP laminate area, f_{fe} is the effective tensile strength allowable to the SRP reinforcement, d_f is the depth of the SRP reinforcement, and s_f is the SRP spacing.

Table 29 summarizes the achieved shear capacity at support for both interior and exterior girders as a function of the adopted strengthening scheme.

Figure 35 shows the as built and strengthened shear capacities compared to the shear demand for all girders.

Table 29 – Girders Shear Capacity at Support (SRP)

FRP Type	Span	Girder Type	Description	κ_v (-)	ϕV_n (kip)	V_u (kip)
Type-5	3-4	Interior	No FRP	-	100.4	171.8
			3 Plies Continuous, U-wrap	0.148	163.2	
		Exterior	No FRP	-	99.5	132.7
			1 Ply Continuous, U-wrap	0.268	134.6	

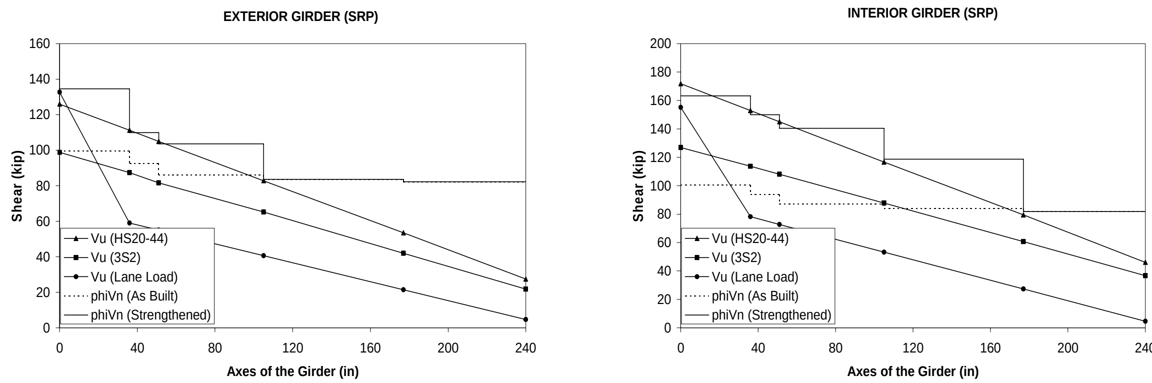


Figure 35 – Shear Demand and Shear Capacity (SRP)

A sketch showing the layout of SRP shear reinforcement is presented in Figure 36. A ½" minimum corner radius needs to be provided when SRP sheet is wrapped around outside corners

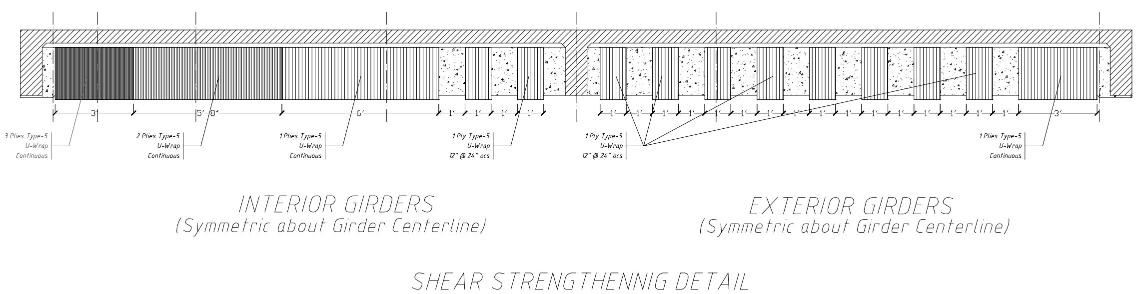


Figure 36 – SRP Shear Strengthening

C.4 Bent Design

C.4.1 Assumptions

Bent geometrical properties are summarized in Figure 37 and Table 30.

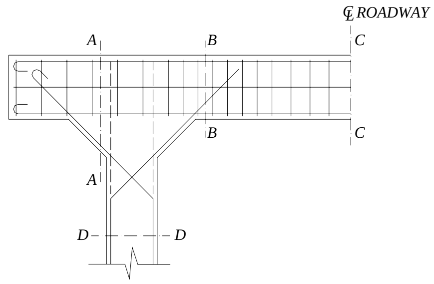


Figure 37 – Portion of the Bent

Table 30 – Bent Geometrical Properties

Member Type	Section	Overall Height h (in)	Width b (in)	Steel Area A_s (in^2)	Effective Depth d (in)	Steel Area A'_s (in^2)	Effective Depth d' (in)	Stirrups Area A_v (in^2)	Stirrups Spacing s (in)
Beam	A-A	48	32	3.95	27.5	3.95	3.0	0.40	12
	B-B	30	32	3.95	27.5	3.95	3.0	0.40	12
	C-C	30	32	3.95	27.5	3.95	3.0	0.40	12
Pier	D-D	24	24	1.32	21.5	1.32	21.5	0.22	12

C.4.2 Positive Moment Strengthening

The beam flexural capacity is summarized in Table 31. Flexural strengthening is needed for the positive moment region of the beam of the bent cup. The bond dependant coefficient, κ_m, has been set equal to *0.9*, and therefore externally bonded FRP U-wraps are deemed necessary to avoid cover delamination or debonding of the flexural laminate.

Table 31 – Positive Moment Flexural Capacity

Section (Figure 37)	Strengthening Description	κ_m (-)	ϕM_n (k-ft)	M_u (k-ft)
C-C	No FRP	-	338.3	892.0
	4 Plies 30” wide	0.9	906.6	

C.4.3 Negative Moment Check

The beam does not need strengthening in its negative moment region. The as-built flexural capacity and flexural demand are summarized in Table 32.

Table 32 – Negative Moment Flexural Capacity

Section (Figure 37)	Strengthening Description	ϕM_n (k-ft)	M_u (k-ft)
A-A	No FRP	338.3	204.1

It is to be noted that the negative moment check has been done using the ultimate moment of cross-section A-A and the geometrical properties and steel reinforcement lay-out of cross-section B-B of Figure 37. The check is conservative.

C.4.4 Shear Capacity Check

The shear capacity of the bent cup is summarized in Table 33.

Table 33 – Beam Shear Capacity

Section (Figure 37)	Strengthening Description	ϕV_n (kip)	V_u (kip)
A-A	No FRP	154.9	162.5

Once again, the shear check has been done assuming the ultimate shear of cross-section A-A and the geometrical properties and steel reinforcement lay-out of cross-section B-B of Figure 37.

Because four plies of FRP laminates need to be bonded as flexural strengthening, two CFRP plies will be installed as U-wraps as shown in Figure 38 and Figure 39 as anchors. The U-wrap laminates will also contribute as shear reinforcement to bridge the small gap between nominal and ultimate shear capacities shown in Table 33.

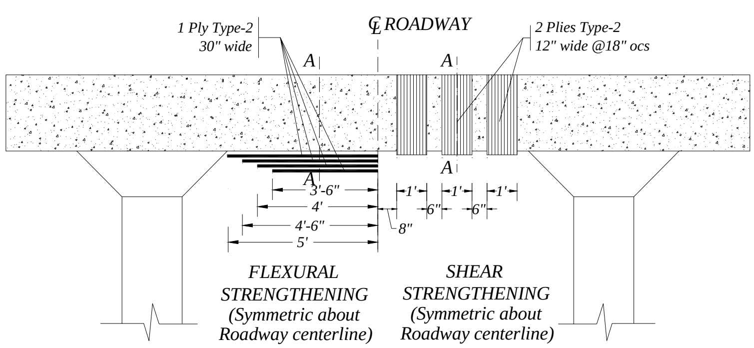


Figure 38 – Strengthening of the Bent

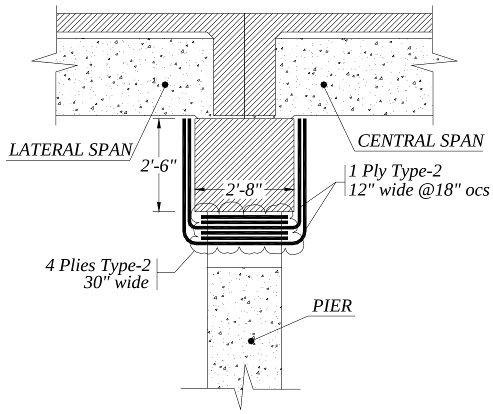


Figure 39 – Cross-Section A-A

C.4.5 Piers Check

Flexural and axial load capacities of the piers need not to be upgraded because the ultimate moment and axial load demand is inside the P-M diagram of the members as shown in Figure 40. Values of ultimate axial load and bending moment are P_u*=241.6 kip* and M_u*=181.7 k-ft*, respectively.

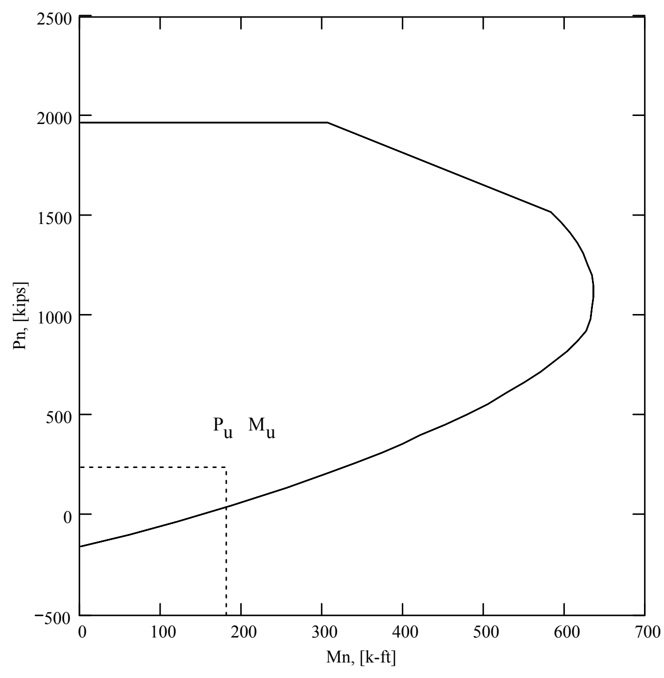


Figure 40 – Pier Flexural and Axial Load Capacity

D. LOAD RATING

Bridge load rating calculations provide a basis for determining the safe load carrying capacity of a bridge. According to MoDOT, anytime a bridge is built, rehabilitated, or reevaluated for any reason, inventory and operating ratings are required using the Load Factor rating method. All bridges should be rated at two load levels, the maximum load level called the Operating Rating and a lower load level called the Inventory Rating. The Operating Rating is the maximum permissible load that should be allowed on the bridge. Exceeding this level could damage the bridge. The Inventory Rating is the load level the bridge can carry on a daily basis without damaging the bridge.

In Missouri, for the Load Factor Method the Operating Rating is based on the appropriate ultimate capacity using current AASHTO specifications (AASHTO, 1994). The Inventory Rating is taken as 86% of the Operating Rating.

The method for determining the rating factor is that outlined by AASHTO in the Manual for Condition Evaluation of Bridges (AASHTO, 1996). Equation (16) was used:

$$RF = \frac{C - A_1 D}{A_2 L\left(1+I\right)} \tag{16}$$

where: RF is the Rating Factor, C is the capacity of the member, D is the dead load effect on the member, L is the live load effect on the member, I is the impact factor to be used with the live load effect, A_1 is the factor for dead loads, and A_2 is the factor for live loads. Since the load factor method is being used, A_1 is taken as 1.3 and A_2 varies depending on the desired rating level. For Inventory rating, $A_2 = 2.17$, and for Operating Rating, $A_2 = 1.3$.

To determine the rating (RT) of the bridge Equation (17) was used:

$$RT = \left(RF\right)W \tag{17}$$

In the above equation, W is the weight of the nominal truck used to determine the live load effect.

For Bridge P-0962, the Load Rating was calculated for a number of different trucks, HS20, H20, 3S2, and MO5. The different ratings are used for different purposes by the bridge owner. For each of the different loading conditions, the maximum shear and maximum moment were calculated. An impact factor is also taken into account for load rating. This value is 29% for Bridge P-0962. The live load effect of each truck on the different elements of the bridge was determined using the same methodology already described in the APPENDICES A-D.

D.1 Slab Rating

The shear and positive and negative moment values for the slab are shown below in Table 34.

Table 34 - Maximum Shear and Positive and Negative Moments due to Live Loads for the Slab

Truck	Maximum Positive Moment [kip-ft/ft]	Maximum Negative Moment [kip-ft/ft]	Maximum Shear [kip/ft]	Maximum Positive Moment with Impact Factor [kip-ft/ft]	Maximum Negative Moment with Impact Factor [kip-ft/ft]	Maximum Shear with Impact Factor [kip/ft]
HS20	4.07	1.78	2.80	5.26	2.30	3.61
MO5	2.04	0.89	1.40	2.63	1.15	3.02
H20	3.40	1.49	2.34	4.39	1.92	3.02
3S2	3.40	1.49	2.34	4.39	1.92	3.02

Table 35 and

Table **36** give the results of the Load Rating pertaining to positive and negative moments respectively, while

Table **37** shows the results for the shear forces.

Table 35 - Rating Factor for the Slab (Positive Bending Moments)

Truck	Rating Factor (RF)	Rating (RT) (Tons)	Rating Type
HS20	1.685	60.6	Operating
HS20	1.009	36.3	Inventory
MO5	3.369	121.3	Operating
H20	1.735	34.7	Posting
3S2	1.735	63.6	Posting

* All Units Expressed in English System

Table 36 - Rating Factor for the Slab (Negative Bending Moments)

Truck	Rating Factor (RF)	Rating (RT) (Tons)	Rating Type
HS20	1.570	56.5	Operating
HS20	0.940	33.9	Inventory
MO5	3.139	113.0	Operating
H20	1.617	32.3	Posting
3S2	1.617	59.2	Posting

* All Units Expressed in English System

Table 37 - Rating Factor for the Slab (Shear)

Truck	Rating Factor (RF)	Rating (RT) (Tons)	Rating Type
HS20	1.414	50.9	Operating
HS20	0.847	30.5	Inventory
MO5	2.829	103.6	Operating
H20	1.457	29.1	Posting
3S2	1.457	53.4	Posting

* All Units Expressed in English System

D.2 Girders Rating

The bending moment values due to the live loads for the exterior girders in corresponding to the most critical sections are summarized below in Table 38.

Table 39 and

Table 40 summarize the corresponding rating factors for the exterior girders reinforced with CFRP laminates and NSM bars and the ones reinforced with SRP respectively.

Table 38 - Maximum Bending Moments due to the Live Loads at the Critical Positions for the Exterior Girders

	Bending Moment at the Critical Positions [kip-ft]				Bending Moment at the Critical Positions with Impact Factor [kip-ft]			
Position [in]	**HS20**	**MO5**	**H20**	**3S2**	**HS20**	**MO5**	**H20**	**3S2**
51	101.6	85.3	68.9	68.5	131.1	110.0	88.9	88.3
105	173.7	145.5	122.0	117.9	224.1	187.7	157.4	152.2
141	210.3	173.1	146.1	141.7	271.3	223.3	188.4	182.8
195	224.6	201.8	165.4	159.5	289.7	260.3	213.4	205.7

240	226.4	211.0	169.3	163.4	292.1	272.2	218.4	210.8

Table 39 – Rating Factors for the Exterior Girders Reinforced with CFRP Laminates and NSM Bars (Bending Moments)

Position [in]	Un-factored Moment due to Dead Load [k-ft]	Moment Capacity [k-ft]	Rating Factors RF_i Computed at the Critical Positions				
			HS20	**HS20**	**MO5**	**H20**	**3S2**
51	140.7	511	2.224	1.154	2.295	2.442	2.457
105	253.2	791.3	1.941	0.950	1.894	1.943	2.009
141	307.4	1127.7	2.422	1.237	2.508	2.556	2.634
195	357.4	1127.7	2.219	1.055	1.960	2.055	2.132
240	370.4	1157	2.272	1.066	1.909	1.893	1.893
Rating Factor: $RF = \min\{RF_i\}$			1.941	0.950	1.894	1.893	1.893
Rating (RT) [Tons]			69.89	34.22	69.39	37.87	69.38
Rating Type			Operating	Inventory	Operating	Posting	Posting

* All Units Expressed in English System

Table 40 – Rating Factors for the Exterior Girders Reinforced with SRP (Bending Moments)

Position [in]	Un-factored Moment due to Dead Load [k-ft]	Moment Capacity [k-ft]	Rating Factors RF_i Computed at the Critical Positions				
			HS20	**HS20**	**MO5**	**H20**	**3S2**
51	140.7	511	2.224	1.154	2.295	2.442	2.457
105	253.2	720.2	1.697	0.804	1.602	1.644	1.700
141	307.4	1059.4	2.229	1.121	2.273	2.316	2.387
195	357.4	1173.9	2.341	1.128	2.096	2.199	2.281
240	370.4	1173.9	2.317	1.092	1.957	1.941	1.941
Rating Factor: $RF = \min\{RF_i\}$			1.697	0.804	1.602	1.644	1.700
Rating (RT) [Tons]			61.10	28.95	58.71	32.87	62.30
Rating Type			Operating	Inventory	Operating	Posting	Posting

* All Units Expressed in English System

The shear force values due to the live loads for the exterior girders in corresponding to the most critical sections are summarized below in Table 41. Table 42 and

Table 43 summarize the corresponding rating factors for the exterior girders reinforced with CFRP laminates and NSM bars and the ones reinforced with SRP respectively.

Table 41 - Maximum Shear Forces at the Critical Positions due to Live Loads for the Exterior Girders

	Shear Forces at the Critical Positions [kip]				Shear Forces at the Critical Positions with Impact Factor [kip]			
Position [in]	**HS20**	**MO5**	**H20**	**3S2**	**HS20**	**MO5**	**H20**	**3S2**
0	27.8	23.0	17.0	17.1	35.9	29.6	21.9	22.1
36	25.1	21.2	16.8	17.0	32.4	27.3	21.7	21.9
51	23.9	19.8	16.2	16.3	30.8	25.6	20.9	21.1
105	19.9	16.3	13.9	14.0	25.7	21.0	18.0	18.1
177	14.6	13.0	10.9	11.0	18.8	16.7	14.1	14.2
240	9.8	9.6	8.3	8.3	12.6	12.4	10.7	10.8

Table 42 - Rating Factors for the Exterior Girders Reinforced with CFRP Laminates and NSM Bars (Shear Forces)

			Rating Factors RF_i Computed at the Critical Positions				
Position [in]	Un-factored Shear Forces due to Dead Load [kip]	Shear Capacity [kip]	**HS20**	**HS20**	**MO5**	**H20**	**3S2**
0	37.0	27.8	1.876	0.970	1.942	1.670	1.670
36	31.5	25.1	2.028	1.097	2.169	2.347	2.330
51	29.2	23.9	1.969	1.077	2.165	2.279	2.263
105	20.8	19.9	2.191	1.292	2.632	2.647	2.628
177	9.7	14.6	3.268	2.114	3.979	4.056	4.028
240	0.0	9.8	5.249	3.609	6.155	6.133	6.090
Rating Factor: $RF = \min\{RF_i\}$			1.876	0.970	1.942	1.670	1.670
Rating (RT) [Tons]			67.54	34.93	71.17	33.41	61.20
Rating Type			Operating	Inventory	Operating	Posting	Posting

* All Units Expressed in English System

Table 43 - Rating Factors for the Exterior Girders Reinforced with SRP (Shear Forces)

			Rating Factors RF_i Computed at the Critical Positions				
Position [in]	Un-factored Shear Forces due to Dead Load [kip]	Shear Capacity [kip]	**HS20**	**HS20**	**MO5**	**H20**	**3S2**
0	37.0	134.6	2.112	1.112	2.225	1.914	1.914
36	31.5	109.9	1.836	0.981	1.941	2.100	2.085
51	29.2	103.6	1.810	0.981	1.973	2.077	2.062
105	20.8	83.6	1.730	1.015	2.068	2.080	2.066
177	9.7	82.2	2.582	1.703	3.205	3.267	3.245
240	0.0	82.2	4.226	2.996	5.110	5.092	5.057
Rating Factor: $RF = \min\{RF_i\}$			1.730	0.981	1.941	1.914	1.914
Rating (RT) [Tons]			62.28	35.32	71.12	38.28	70.12
Rating Type			Operating	Inventory	Operating	Posting	Posting

* All Units Expressed in English System

The bending moment values due to the live loads for the interior girders corresponding to the most critical sections are summarized below in Table 44.

Table 45 and

Table **46** summarize the corresponding rating factors for the interior girders reinforced with CFRP laminates and NSM bars and the ones reinforced with SRP respectively.

Table 44 - Maximum Bending Moments due to the Live Loads at the Critical Positions for the Interior Girders

	Bending Moment at the Critical Positions [kip-ft]				Bending Moment at the Critical Positions with Impact Factor [kip-ft]			
Position [in]	**HS20**	**MO5**	**H20**	**3S2**	**HS20**	**MO5**	**H20**	**3S2**
51	172.0	144.3	116.6	115.9	221.9	186.1	150.4	149.5
105	294.0	246.2	206.4	199.6	379.3	317.6	266.3	257.5

141	340.7	292.9	247.2	239.8	439.5	377.8	318.9	309.4
195	380.1	341.4	280.0	269.9	490.3	440.5	361.1	348.1
240	383.1	357.0	286.4	276.5	494.2	460.5	369.5	356.7

Table 45 – Rating Factors for the Interior Girders Reinforced with CFRP Laminates and NSM Bars (Bending Moments)

			Rating Factors RF_i Computed at the Critical Positions				
Position [in]	Un-factored Moment due to Dead Load [k-ft]	Moment Capacity [k-ft]	**HS20**	**HS20**	**MO5**	**H20**	**3S2**
51	120.5	172	1.256	0.892	1.774	1.888	1.900
105	216.8	294	1.494	1.017	2.027	2.080	2.151
141	263.2	340.7	1.550	1.034	2.008	2.046	2.109
195	306.0	380.1	1.426	0.945	1.755	1.841	1.910
240	317.2	383.1	1.523	0.992	1.778	1.905	1.974
Rating Factor: $RF = \min\{RF_i\}$			1.256	0.892	1.755	1.841	1.900
Rating (RT) [Tons]			45.22	32.09	64.31	36.82	69.61
Rating Type			Operating	Inventory	Operating	Posting	Posting

* All Units Expressed in English System

Table 46 – Rating Factors for the Interior Girders Reinforced with SRP (Bending Moments)

			Rating Factors RF_i Computed at the Critical Positions				
Position [in]	Un-factored Moment due to Dead Load [k-ft]	Moment Capacity [k-ft]	**HS20**	**HS20**	**MO5**	**H20**	**3S2**
51	120.5	521.1	1.031	0.757	1.506	1.603	1.613
105	216.8	1069.1	1.393	0.957	1.907	1.956	2.023
141	263.2	1279.5	1.464	0.983	1.908	1.945	2.004
195	306.0	1452.9	1.504	0.992	1.843	1.933	2.005

240	317.2	1452.9	1.486	0.970	1.738	1.863	1.930
Rating Factor: $RF = \min\{RF_i\}$			1.031	0.757	1.506	1.603	1.613
Rating (RT) [Tons]			37.13	27.25	55.19	32.07	59.10
Rating Type			Operat-ing	Inven-tory	Operat-ing	Posting	Posting

* All Units Expressed in English System

The shear force values due to the live loads for the interior girders corresponding to the most critical sections are summarized below in

Table **47**. Table 48 and

Table **49** summarize the corresponding rating factors for the interior girders reinforced with CFRP laminates and NSM bars and the ones reinforced with SRP respectively.

Table 47 - Maximum Shear Forces due to Live Loads at the Critical Positions for the Interior Girders

	Shear Forces at the Critical Positions [kip]				Shear Forces at the Critical Positions with Impact Factor [kip]			
Position [in]	**HS20**	**MO5**	**H20**	**3S2**	**HS20**	**MO5**	**H20**	**3S2**
0	47.0	38.9	28.8	32.2	60.6	50.2	37.1	41.5
36	42.2	35.9	28.5	29.2	54.4	46.2	36.8	37.6
51	40.5	33.6	27.4	26.9	52.3	43.3	35.4	34.7
105	33.6	27.6	23.6	22.7	43.3	35.6	30.4	29.2
177	24.7	21.9	18.5	17.8	31.9	28.3	23.8	22.9
240	16.6	16.2	14.0	12.9	21.4	20.9	18.1	16.6

Table 48 - Rating Factors for the Interior Girders Reinforced with CFRP Laminates and NSM Bars (Shear Forces)

			Rating Factors RF_i Computed at the Critical Positions				
Position [in]	Un-factored Shear Forces due to Dead Load [kip]	Shear Capacity [kip]	**HS20**	**HS20**	**MO5**	**H20**	**3S2**
0	31.7	164.2	1.308	0.935	1.887	2.019	1.960
36	27.0	158.0	1.457	1.040	2.044	2.212	2.162
51	25.0	148.0	1.404	1.019	2.051	2.160	2.203
105	17.8	118.0	1.319	1.009	2.050	2.062	2.147
177	8.3	90.0	1.398	1.146	2.156	2.198	2.285

240	0.0	90.0	2.458	1.937	3.307	3.295	3.580
Rating Factor: $RF = \min\{RF_i\}$			1.308	0.935	1.887	2.019	1.960
Rating (RT) [Tons]			47.09	33.65	69.12	40.38	71.80
Rating Type			Operat-ing	Inven-tory	Operat-ing	Posting	Posting

* All Units Expressed in English System

Table 49 - Rating Factors for the Interior Girders Reinforced with SRP (Shear Forces)

			Rating Factors RF$_i$ Computed at the Critical Positions				
Position [in]	Un-factored Shear Forces due to Dead Load [kip]	Shear Capacity [kip]	**HS20**	**HS20**	**MO5**	**H20**	**3S2**
0	31.7	163.2	1.295	0.927	1.871	2.003	1.944
36	27.0	150.0	1.344	0.973	1.911	2.068	2.021
51	25.0	140.4	1.292	0.952	1.916	2.017	2.058
105	17.8	118.7	1.331	1.016	2.065	2.077	2.163
177	8.3	81.9	1.202	1.028	1.935	1.973	2.051
240	0.0	81.9	2.167	1.762	3.009	2.998	3.258
Rating Factor: $RF = \min\{RF_i\}$			1.202	0.927	1.871	1.973	1.944
Rating (RT) [Tons]			43.27	33.38	68.56	39.46	71.21
Rating Type			Operat-ing	Inven-tory	Operat-ing	Posting	Posting

* All Units Expressed in English System

D.3 Bents

Table **50** summarizes the vertical forces due to the live loads transferred to the bents corresponding to the possible load combinations of the girders.

Table **51** summarizes ultimate moments and shear forces calculated at four cross-sections where maximum values are reached.

Table 50 – Load to the Bent due to the Live Load (Including Impact Factors)

Loading Conditions	Force to the Bent [kip]	HS-20	H-20	MO-5	3S2
Ia)	P_{1L}	37.152	29.617884	46.6845	35.463519
	P_L	40.506	32.264964	50.8569	38.633049
	P_{2L}	-3.87	-3.08826	-4.8678	-3.697785
Ib)=IIa)	P_{1L}	21.93	17.441316	27.4915	20.883681
	P_L	54.696	43.559172	68.6592	52.156377
	P_{2L}	-2.709	-2.176488	-3.43064	-2.606058
IIb)	P_{1L}	5.418	4.382388	6.90764	5.247333
	P_L	62.823	50.118048	78.9974	60.009768
	P_{2L}	5.418	4.382388	6.90764	5.247333

Table 51 – Bending Moments and Shear Forces at the Critical Cross Sections

	Bending Moment at the Critical Positions [kip-ft/ft]				Shear Forces at the Critical Positions [kip-ft/ft]			
Sections	**HS20**	**H20**	**3S2**	**MO5**	**HS20**	**H20**	**3S2**	**MO5**
AA	68.1	65.0	85.6	54.3	37.2	29.6	35.5	46.7
BB	87.1	87.1	99.6	87.1	42.4	42.4	42.4	42.4
CC	230.7	230.7	230.7	230.7	42.4	42.4	42.4	42.4
DD	46.2	46.2	46.2	46.2	4.6	4.6	4.6	4.6

Table 52 - Rating Factors for the Bents (Bending Moments)

			Rating Factors RF_i Computed at the Critical Positions				
Sections	Un-factored Bending Moment due to Dead Load [k-ft]	Moment Capacity [k-ft]	**HS20**	**HS20**	**MO5**	**H20**	**3S2**
AA	43.3	338.3	3.185	1.908	2.535	3.436	2.870
BB	106.0	338.3	1.771	1.061	1.549	1.523	1.523
CC	300.9	906.6	1.718	1.029	1.718	1.478	1.478
DD	62.7	327.3	4.094	2.452	4.094	3.520	3.520
Rating Factor: $RF = \min\{RF_i\}$			1.718	1.029	1.549	1.478	1.478
Rating (RT) [Tons]			61.86	37.06	56.77	29.55	54.14
Rating Type			Operating	Inventory	Operating	Posting	Posting

* All Units Expressed in English System

Table 53 - Rating Factors for the Bents (Shear Forces)

Sections	Un-factored Shear Forces due to Dead Load [kip]	Shear Capacity [kip]	Rating Factors RF_i Computed at the Critical Positions				
			HS20	**HS20**	**MO5**	**H20**	**3S2**
AA	23.6	154.9	2.572	1.541	2.047	2.774	2.317
BB	54.3	154.9	1.532	0.918	1.532	1.317	1.317
CC	54.3	154.9	1.532	0.918	1.532	1.317	1.317
DD	6.3	86.3	13.018	7.799	13.018	11.195	11.195
Rating Factor: $RF = \min\{RF_i\}$			1.532	0.918	1.532	1.317	1.317
Rating (RT) [Tons]			55.14	33.03	56.12	26.34	48.26
Rating Type			Operating	Inventory	Operating	Posting	Posting

* All Units Expressed in English System

D.4 Piers

Axial loads due to live loads and corresponding rating factors are summarized in Table 54 and Table 55.

Table 54 - Axial Loads due to Live Loads

Truck	Maximum Axial Load [kip]	Maximum Axial Load with Impact Factor [kip]
HS20	4.07	5.26
MO5	2.04	2.63
H20	3.40	4.39
3S2	3.40	4.39

Table 55 - Rating Factor for the Piers (Axial Loads)

Truck	Rating Factor (RF)	Rating (RT) (Tons)	Rating Type
HS20	12.877	463.6	Operating
HS20	7.714	277.7	Inventory
MO5	12.877	463.6	Operating
H20	11.074	221.5	Posting
3S2	11.074	405.8	Posting

* All Units Expressed in English System

D.5 Summary and Conclusions

The rating of the bridge is determined by the least rated element. Table 56 summarizes the rating of each element of the bridge. The most deficient elements are the exterior girders strengthened with SRP.

Since the factors RF with which posting is determined are greater than 1 the bridge does not need to be load posted. In addition, from Table 56 the maximum operating and inventory load can be found as 37.1 T and 23.3 T respectively.

Table 56 – Summary of the rating of all the elements

	Rating Factors RF_E for the Elements				
Elements	**HS20**	**HS20**	**MO5**	**H20**	**3S2**
Slab (Positive Bending Moments)	1.685	1.009	3.369	1.735	1.735
Slab (Negative Bending Moments)	1.570	0.940	3.139	1.617	1.617
Slab (Shear Forces)	1.414	0.847	2.829	1.457	1.457
Exterior Girders CFRP Laminates and NSM Bars (Bending Moments)	1.941	0.950	1.894	1.893	1.893
Exterior Girders CFRP Laminates and NSM Bars (Shear Forces)	1.697	0.804	1.602	1.644	1.700
Interior Girders CFRP Laminates and NSM Bars (Bending Moments)	1.876	0.970	1.942	1.670	1.670
Interior Girders CFRP Laminates and NSM Bars (Shear Forces)	1.730	0.981	1.941	1.914	1.914
Exterior Girders SRP (Bending Moments)	1.256	0.892	1.755	1.841	1.900
Exterior Girders SRP (Shear Forces)	1.031	0.757	1.506	1.603	1.613
Interior Girders SRP (Bending Moments)	1.308	0.935	1.887	2.019	1.960
Interior Girders SRP (Shear Forces)	1.202	0.927	1.871	1.973	1.944
Bents (Bending Moments)	1.718	1.029	1.549	1.478	1.478
Bents (Shear Forces)	1.532	0.918	1.532	1.317	1.317
Piers	12.877	7.714	12.877	11.074	11.074
Rating Factor: $RF = \min\{RF_E\}$	1.031	0.757	1.506	1.317	1.317
Rating (RT) [Tons]	37.12	27.25	55.18	26.34	48.25
Rating Type	Operating	Inventory	Operating	Posting	Posting

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[1] AASHTO, 2002: "Standard Specifications for Highway Bridges", 17th Edition, Published by the American Association of State Highway and Transportation Officials, Washington D.C.

[2] ACI 440.2R-02, 2002: "Guide for the Design and Construction of Externally Bonded FRP Systems for Strengthening Concrete Structures," Published by the American Concrete Institute, Farmington Hills, MI.

[3] AASHTO, 1998: "LRFD Bridge Design Specifications", Second Edition, Published by the American Association of State Highway and Transportation Officials, Washington D.C.

[4] ACI 318-99, 1999: "Building Code Requirements for Structural Concrete and Commentary (318R-99)," Published by the American Concrete Institute, Farmington Hills, MI.

[5] AASHTO, 1996 "LRFD Design Code for Highway Bridges", Published by the American Association of State Highway and Transportation Officials, Washington, D.C.

[6] AASHTO, 1994 "Manual for Condition Evaluation of Bridges", Published by the American Association of State Highway and Transportation Officials, Washington, D.C.

APPENDIX A – Load Transfer and Slab Analysis

The statically indeterminate structure shown in Figure 7-*I* can be reduced to two simpler structures as represented in Figure 41. The vertical reaction R_2 represents the unknowns of the problem to be determined by imposing the compatibility of the displacements as expressed in Eq. (18).

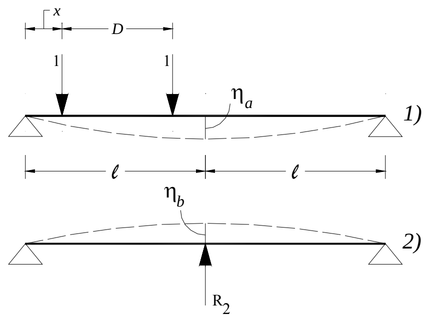

Figure 41 – Structures Equivalent to Figure 7-*I*

$$\begin{aligned} \eta_a &= \eta_b \\ \eta_b &= \frac{R_2 \ell^3}{6EI} \end{aligned} \tag{18}$$

By using superposition, Beam 1 in Figure 41 is equivalent to the two beams shown in Figure 42. The compatibility equation can be rearranged as follows:

$$\begin{aligned} \eta_a &= \eta_{a-1} + \eta_{a-2} \\ \eta_{a-1} &= \frac{x}{12EI}\left(3\ell^2 - x^2\right) \\ \eta_{a-2} &= \frac{x+D}{12EI}\left(3\ell^2 - (x+D)^2\right) \end{aligned} \tag{19}$$

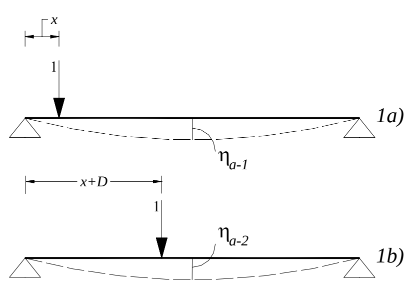

Figure 42 – Structures Equivalent to Beam 1 in Figure 41

The unknown R_2 can be determined as follows:

$$R_2 = \frac{6}{\ell^3}(\eta_{a-1} + \eta_{a-2}) \tag{20}$$

Bending moment and shear force can be found from the following Eq. (21) (see Figure 43). It should be noted that the vertical reactions from Eq. (20) needs to be multiplied by *P/2 (P=axle load)* because the previous analysis was conducted using unit forces.

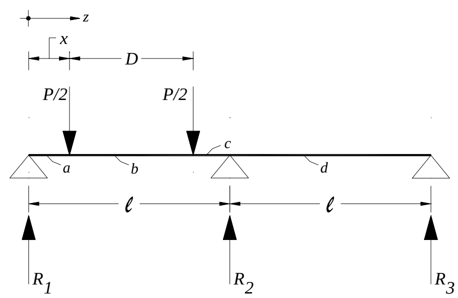

Figure 43 – Definitions for M and V

$$\begin{aligned}
M_a &= R_1 z \\
M_b &= M_a - 0.5P(z-x) \\
M_c &= M_b - 0.5P[z-(x+D)] \\
M_d &= M_c + R_2(z-\ell) \\
V_a &= R_1 \\
V_b &= V_a - 0.5P \\
V_c &= V_b - 0.5P \\
V_d &= V_c + R_2
\end{aligned} \tag{21}$$

The case shown in Figure 7-*II* is similar to the one already presented if:

$$\begin{aligned}
\eta_a &= \eta_{a-1} + \eta_{a-2} \\
\eta_{a-1} &= \frac{x}{12EI}\left(3\ell^2 - x^2\right) \\
\eta_{a-2} &= \frac{2\ell-(x+D)}{12EI}\left(3\ell^2 - [2\ell-(x+D)]^2\right)
\end{aligned} \tag{22}$$

Bending moment and shear force for this second case can be written as follows:

$$
\begin{aligned}
&M_a = R_1 z \\
&M_b = M_a - 0.5P(z - x) \\
&M_c = M_b + R_2(z - \ell) \\
&M_d = M_c - 0.5P[z - (x + D)] \\
&V_a = R_1 \\
&V_b = V_a - 0.5P \\
&V_c = V_b - R_2 \\
&V_d = V_c - 0.5P
\end{aligned}
\tag{23}
$$

APPENDIX B – Load Lane Analysis

a) Distributed Load

As stated in AASHTO, the load lane load consists of *q=0.64 kip/ft*, uniformly distributed in the longitudinal direction. Transversely, it is uniformly distributed over a *10.0 ft* width. The share that each girder carries can be found by analyzing the structure shown in Figure 44.

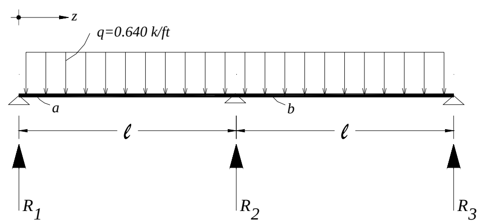

Figure 44 – Determination of k_ℓ

The beam represented in Figure 44 can be analyzed by removing the central support and imposing the compatibility equation. The vertical reactions can be written as follows:

$$
\begin{aligned}
R_1 &= R_3 = \frac{3}{8} q\ell \\
R_2 &= \frac{5}{4} q\ell
\end{aligned}
\tag{24}
$$

The values of k_ℓ are *1.25* and *0.375* for interior and exterior girders, respectively. Bending moment and shear force can be expressed as follows:

$$
\begin{aligned}
M_a &= \frac{3}{8} q\ell z - q\frac{z^2}{2} \\
M_b &= \frac{3}{8} q\ell z - q\frac{z^2}{2} + \frac{5}{4} q\ell (z-\ell) \\
V_a &= \frac{3}{8} q\ell - qz \\
V_b &= \frac{3}{8} q\ell - qz + \frac{5}{4} q\ell
\end{aligned}
\tag{25}
$$

When only half of the deck is loaded with the uniform distribution q, the k_ℓ coefficients are *0.438* and *0.625* for the external loaded reaction and internal reaction, respectively.

APPENDIX C – Girder Analysis for an HS20-44 Truck

As previously recognized, girder analysis is carried out by taking into consideration only three of the five cases corresponding to five different positions of the design truck on the single span, as shown in Figure 13. The first case of Figure 13 is enlarged in Figure 45.

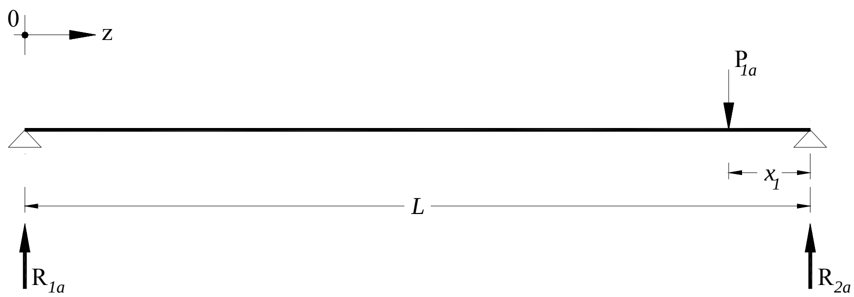

Figure 45 – One Wheel Load on the Girder

Vertical reactions R_1 and R_2 are defined as follows:

$$
\begin{aligned}
R_{1a} &= P_{1a} - R_{2a} \\
R_{2a} &= \frac{P_{1a}(L - x_1)}{L}
\end{aligned}
\tag{26}
$$

Shear and moment diagrams can be expressed as a function of z as follows:

$$
\begin{aligned}
V_a(z) &= \begin{cases} R_{1a} & if \quad z \le L - x_1 \\ R_{1a} - P_{1a} & otherwise \end{cases} \\
M_a(z) &= \begin{cases} R_{1a} z & if \quad z \le L - x_1 \\ R_{1a} z - P_{1a}[z - (L - x_1)] & otherwise \end{cases}
\end{aligned}
\tag{27}
$$

The second case (Figure 13*b)* is shown in Figure 46. Vertical reactions are:

$$
\begin{aligned}
R_{1b} &= P_{1a} + P_{1b} - R_{2b} \\
R_{2b} &= R_{2a} + \frac{P_{1b}(L - x_1 + L_{1a})}{L}
\end{aligned}
\tag{28}
$$

Shear and moments can be written as:

$$V_b(z) = \begin{cases} R_{1b} \quad if \quad z \le L - x_1 \\ R_{1b} - P_{1a} \quad if \quad L - x_1 \le z \le L - x_1 + L_{1a} \\ R_{1b} - P_{1a} - P_{1b} \quad otherwise \end{cases}$$

$$M_b(z) = \begin{cases} R_{1b}z \quad if \quad z \le L - x_1 \\ R_{1b}z - P_{1a}[z-(L-x_1)] \quad if \quad L - x_1 \le z \le L - x_1 + L_{1a} \\ R_{1b}z - P_{1a}[z-(L-x_1)] + P_{1b}[z-(L-x_1+L_{1a})] \quad otherwise \end{cases} \tag{29}$$

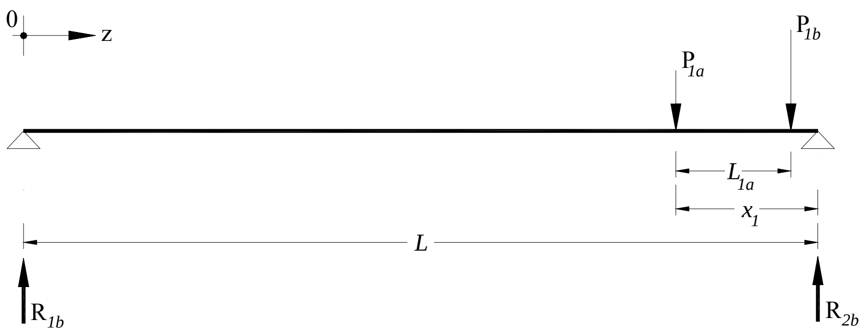

Figure 46 – Two Wheel Loads on the Girder

When three wheel loads are present on the girder (see Figure 47), vertical reactions are expressed as follows:

$$R_{1c} = P_{1a} + P_{1b} + P_{1c} - R_{2c}$$
$$R_{2c} = R_{2b} + \frac{P_{1c}(L - x_1 + L_{1a} + L_{1b})}{L} \tag{30}$$

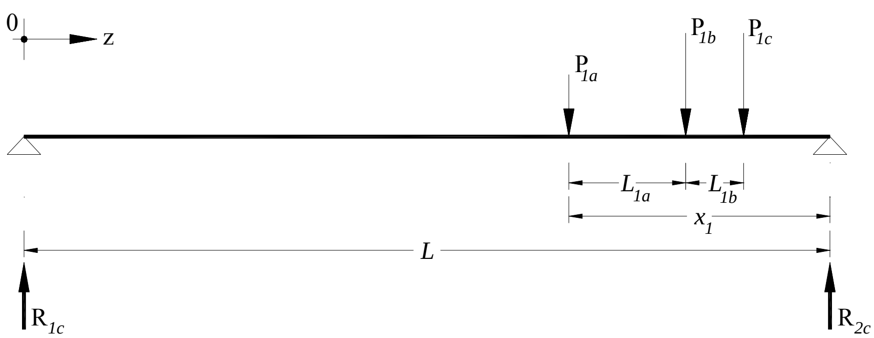

Figure 47 – Three Wheel Loads on the Girder

Shear and moments can be written as follows:

$$V_c(z)=\begin{cases} R_{1c} & if \quad z \le L-x_1 \\ R_{1c}-P_{1a} & if \quad L-x_1 \le z \le L-x_1+L_{1a} \\ R_{1c}-P_{1a}-P_{1b} & if \quad L-x_1+L_{1a} < z < L-x_1+L_{1a}+L_{1b} \\ R_{1c}-P_{1a}-P_{1b}-P_{1c} & otherwise \end{cases}$$

$$M_c(z)=\begin{cases} R_{1c}z \quad if \quad z \le L-x_1 \\ R_{1c}z-P_{1a}[z-(L-x_1)] \quad if \quad L-x_1 \le z \le L-x_1+L_{1a} \\ R_{1c}z-P_{1a}[z-(L-x_1)]-P_{1b}[z-(L-x_1+L_{1a})] \quad if \quad L-x_1+L_{1a} < z < L-x_1+L_{1a}+L_{1b} \\ R_{1c}z-P_{1a}[z-(L-x_1)]-P_{1b}[z-(L-x_1+L_{1a})]-P_{1c}[z-(L-x_1+L_{1a}+L_{1b})] \quad otherwise \end{cases} \tag{31}$$

APPENDIX D – Bent Analysis

The bent represented in Figure 17 can be considered equivalent to the the structures shown in Figure 48.

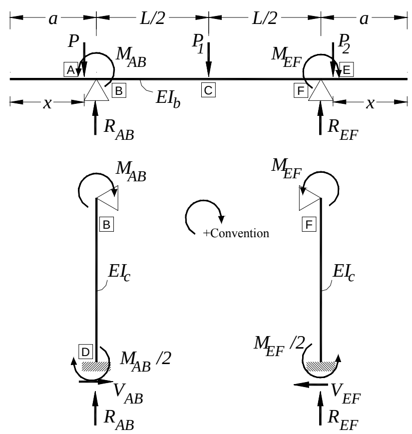

Figure 48 – Bent Equivalent Structures (Live Load)

The compatibility equation can be written as follows:

$$\begin{aligned} \alpha_{B\_beam} &= \alpha_{B\_column} \\ \alpha_{F\_beam} &= \alpha_{F\_column} \end{aligned} \tag{32}$$

where:

$$\begin{aligned} \alpha_{beam} &= \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 \\ \alpha_1 &= \frac{-P(a-x)L}{3EI_b} \\ \alpha_2 &= \frac{-M_{AB}L}{3EI_b} \\ \alpha_3 &= \frac{P_1 L^2}{16EI_b} \\ \alpha_4 &= \frac{-M_{EF}L}{6EI_b} \\ \alpha_5 &= \frac{-P_2(a-x)L}{6EI_b} \end{aligned} \tag{33}$$

and:

$$\alpha_{B\_column} = \frac{M_{AB}h}{4EI_c} \tag{34}$$

Similar equations can be written to meet the compatibility at F (Figure 48).
By resolving the previous equation, the unknown M_{AB} can be determined.