



# PRESERVATION OF MISSOURI TRANSPORTATION INFRASTRUCTURES

## **VOL I: Bridge Design & Load Rating**



VALIDATION OF FRP COMPOSITE TECHNOLOGY  
THROUGH FIELD TESTING

### **Strengthening of Bridge X-0495 Iron County, MO**

*Prepared for:*  
*Missouri Department of Transportation*  
*University of Missouri-Rolla*  
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## A. INTRODUCTION

### A.1 General Description

In the following report, the analysis and design procedures used in the upgrade of the load-posted Bridge X-0495, located in Iron County, MO are summarized. Figure 1 shows a picture of the bridge. The total bridge length is  $137.5\text{ ft}$  and the total width of the deck is  $23.6\text{ ft}$ .



Figure 1 – Bridge X-0495

The structure has three spans and each of them consists of three reinforced concrete (RC) girders monolithically cast with a  $6\text{ in.}$  slab, as depicted in Figure 2. Each span is provided with one transversal beam of the same depth as the main girders. The lateral spans are  $42.5\text{ ft}$  while the central span is  $52.5\text{ ft}$ .



Figure 2 – Superstructure of the Bridge

### A.2 Objectives

The objective of this document is to provide an analysis of the structure and the design calculations for its strengthening using externally bonded fiber-reinforced polymer (FRP) systems. The FRP systems consist of FRP laminates to be installed by manual lay-up to the main longitudinal girders as flexural and shear reinforcement. In addition, near



surface mounted (NSM) FRP bars will also be installed as flexural strengthening. The same strengthening methodology will be used to upgrade the flexural capacity of the slab.

### **A.3 Assumptions**

The following assumptions are made:

- a) Nominal material properties for steel and concrete. At the onset of the project, existing material properties were validated in the field by extracting two concrete cores and steel bar sample. The resulting values are:  $f_c=5450\text{ psi}$ , and  $f_y=40\text{ ksi}$
- b) Load configurations and analysis are consistent with AASHTO<sup>1</sup> Specifications; and
- c) Design of the strengthening system is in compliance with ACI 440.2R-02<sup>2</sup>, where applicable.

## B. STRUCTURAL ANALYSIS

### B.1 Load Combinations

For the structural analysis of the bridge, definitions of the design truck, load lane, and design lane are necessary, as well as the transversal load distribution. These issues will be discussed in the next two sections. A plan view of the bridge piers and abutments is shown in Figure 3.

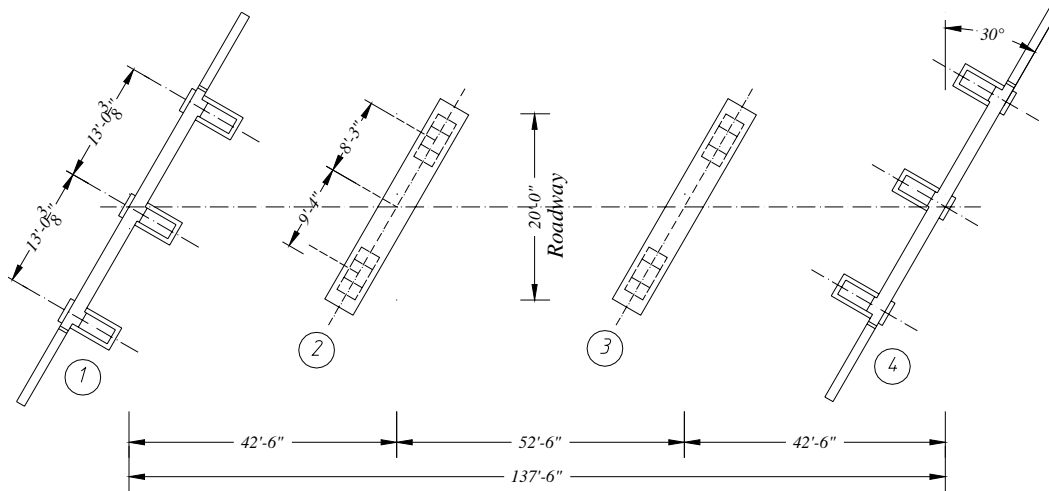


Figure 3 – Plan View of the Bridge (Not to scale)

Ultimate values of bending moment and shear force are obtained by multiplying their nominal values by the dead and live load factors and by the impact factor according to AASHTO Specifications as shown in Eq. (1):

$$\omega_u = 1.3[\beta_d D + 1.67(L + I)] \quad (1)$$

where  $D$  is the dead load,  $L$  is the live load,  $\beta_d = 1.0$  as per AASHTO Table 3.22.1A, and  $I$  is the live load impact calculated as follows:

$$I = \frac{50}{L + 125} \quad (2)$$

and  $L$  represents the span length from center-to-center of support. Table 1 summarizes the values of  $I$  for both lateral and central span of the bridge.

Table 1 – Impact Factor,  $I$

Span	$L$ , (ft)	$I$ , (%)
Central	52.5	28
Lateral	42.5	30

## B.2 Design Trucks, Load Lanes and Design Lanes

The analysis of the bridge is carried out for an HS20-44 truck load (which represents the AASHTO design truck load) and for a 3S2 truck load as requested by MoDOT, having geometrical characteristics and weight properties as shown in Figure 4.

According to AASHTO Section 3.6.3, roadway widths between 20.0 and 24.0 ft shall have two design lanes, each equal to one-half of the roadway width. Although the roadway width of the bridge is 20.0 ft, only one design lane has been considered for the truck load analysis. For the load lane analysis, however, two or one lanes will be considered depending on the worst loading scenario.

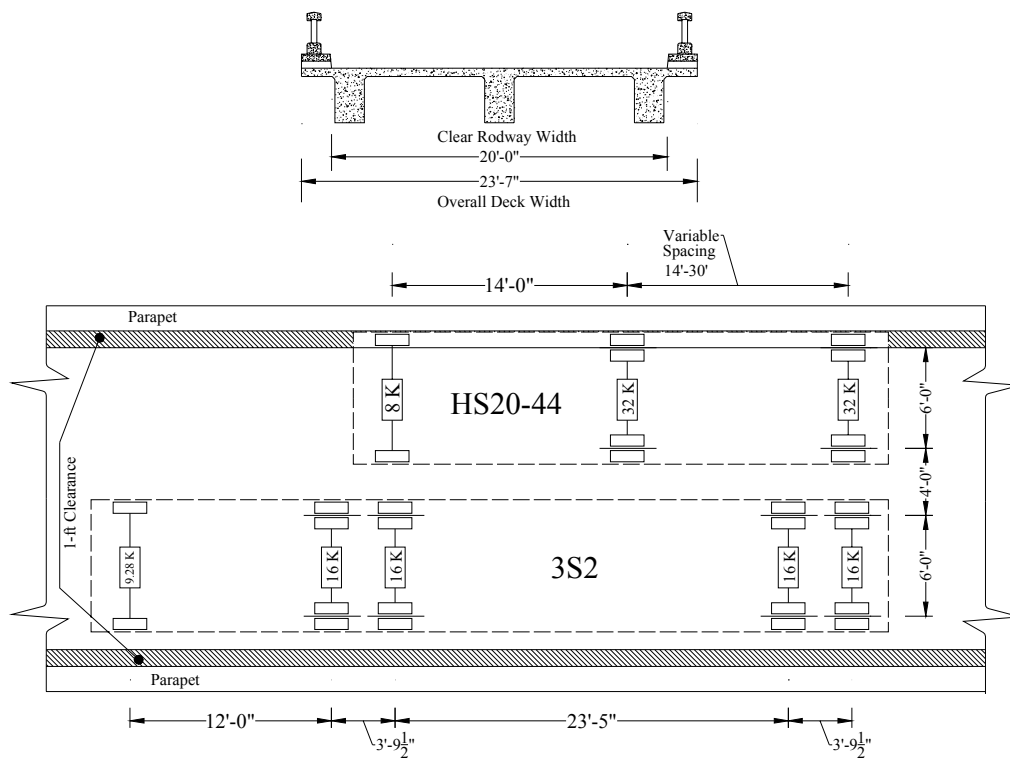


Figure 4 – Truck Load and Truck Lanes

Note that the centerline of the double-wheel of the rear axle shown in Figure 4 is located not less than 1.00 ft away from the curb as specified in AASHTO for slab design.

Three loading conditions are required to be checked as laid out in Figure 5.

The HS20-44 design truck load (Figure 5a) has a front axle load of 8.0 kip, second axle load, located 14.0 ft behind the drive axle, of 32.0 kip, and rear axle load also of 32.0 kip. The rear axle load is positioned at a variable distance, ranging between 14.0 and 30.0 ft. Given the specific bridge geometry, the worst loading scenario is obtained for the minimum spacing of 14.0 ft between the two rear axles.

The 3S2 design truck load has five axles; the front axle of 9.28 kip, the second double axle, located 12.0 ft behind the drive axle, of 16.0 kip, and the rear double axle also of 16.0 kip, as shown in Figure 5b). Distances between axles are given in the figure.

The load lane condition consists of a load of 640 lbs per linear foot, uniformly distributed in the longitudinal direction with a single concentrated load so placed on the span as to produce maximum stress. The concentrated load and uniform load is considered to be uniformly distributed over a 10'-0" width on a line normal to the centerline of the lane. The intensity of the concentrated load is represented in Figure 5c) for both bending moment and shear force calculations. This load shall be placed in such positions within the load lane as to produce the maximum stress in the member.

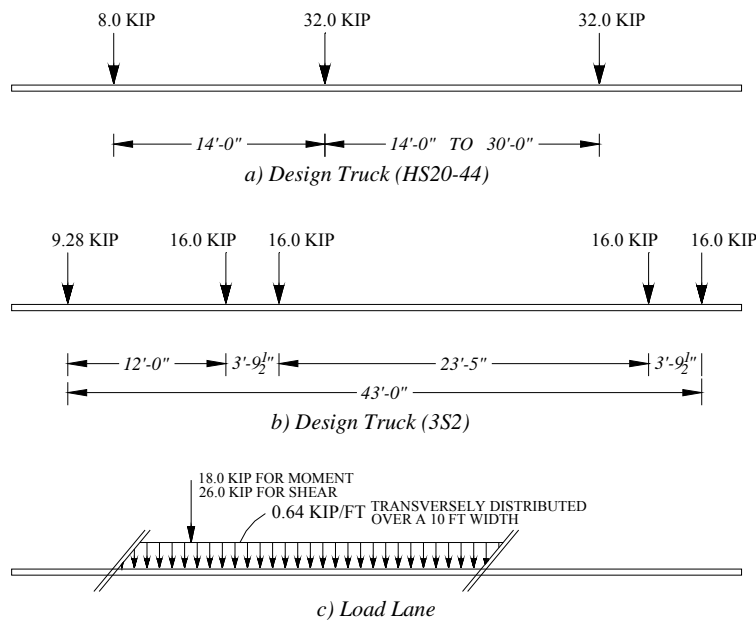


Figure 5 – Loading Conditions

### B.3 Slab Analysis

The deck slab is considered to be a one-way slab system due to its large aspect ratio (panel length divided by the panel width). The width of the slab strip to be used in the analysis and design is provided by AASHTO<sup>3</sup> (Table 4.6.2.1.3-1) and, for cast-in-place

concrete, may be written as follows for positive and negative moment regions, respectively:

$$\begin{aligned} b^+ &= 26.0 + 6.6S \text{ in.} \\ b^- &= 48.0 + 3.0S \text{ in.} \end{aligned} \tag{3}$$

where  $S$  represents the center-to-center spacing of the girders (*ft*).

A generic slab-girder system displaces as shown in Figure 6*a*. This displacement can be seen as the superposition of the displacement associated with local effects represented in Figure 6*b* and the global effect due to the vertical displacement of the girders. Since the local effect is usually significantly greater than the global effect, the latter will be neglected, and the strip analyzed using classical beam theory, assuming that the girders provide rigid support.

The analysis of the slab is carried out on a structure similar to that shown in Figure 6*b*).

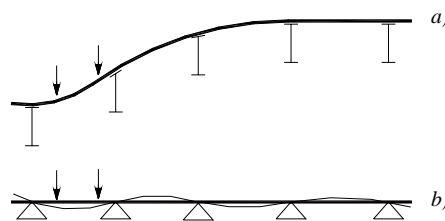


Figure 6 – Slab Deck Deflection due to External Loads

### B.3.1 Results of the Analysis

In the following, only the results will be presented. A detailed protocol for the analysis is shown in APPENDIX A and APPENDIX B.

The four loading conditions being considered are shown in Figure 7. The first two loading conditions, *I*) and *II*), are related to the design truck load. Loading conditions *III*) and *IV*) of Figure 7 refer to the load lane.

Table 2 summarizes the results in terms of ultimate (factored) bending moments and shear forces for the case of wheel loads corresponding to the HS20-44 truck load, which represents the most demanding loading condition. Results of Table 2 take into account the moment and shear due to the asphalt layer (*6 in.*) and the self-weight of the deck. The values are adopted for design.

Figure 8 shows the bending moment diagrams due to the live load only as the design truck load moves transversally on the bridge deck for both loading condition *I*) and *II*) and central and lateral span, respectively. These moments were divided by the strip widths shown in Eq. (3) to obtain the values of unit moment summarized in Table 2. Values of Table 2 take into account the moment due to the dead load as well.

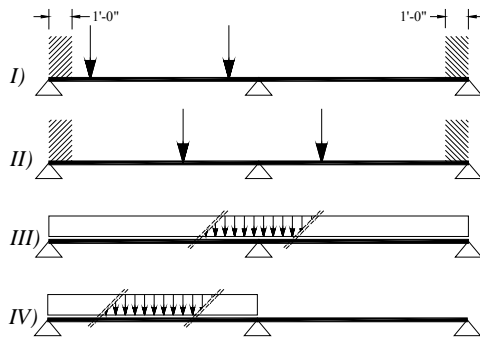


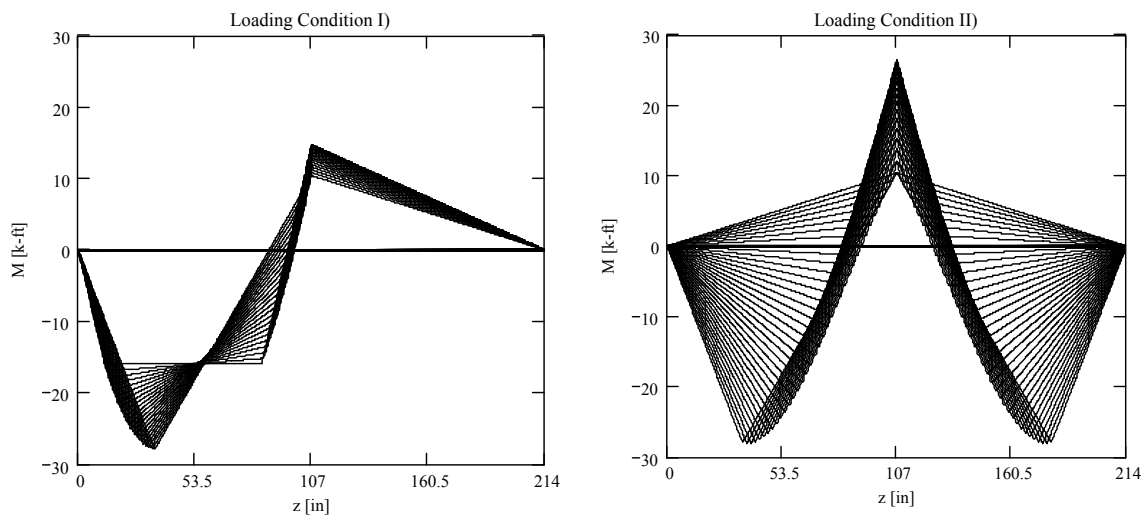
Figure 7 – Loading Conditions for Slab Analysis

Table 2 – Slab Ultimate Bending Moments and Shear Forces per Unit Strip

Span	Span Length (in)	Loading Condition	Number of Design Lanes Considered	Positive Moment <sup>a)</sup> (k-ft/ft)	Negative Moment <sup>b)</sup> (k-ft/ft)	Shear <sup>b)</sup> (kip/ft)
Central	107	I)	1	11.9	6.8	9.2
		II)	1	12.0	7.6	8.1
		III)	2	2.4	2.5	2.1
		IV)	1	2.9	2.2	2.0
Lateral	109	I)	1	12.2	7.4	9.6
		II)	1	12.2	8.8	8.2
		III)	2	2.4	3.0	2.3
		IV)	1	3.0	2.4	2.1

a) Computed close to mid-span depending on load location (See Figure 8)

b) Computed at a cross-section flush with the girder, including 2" chamfer



Central Span

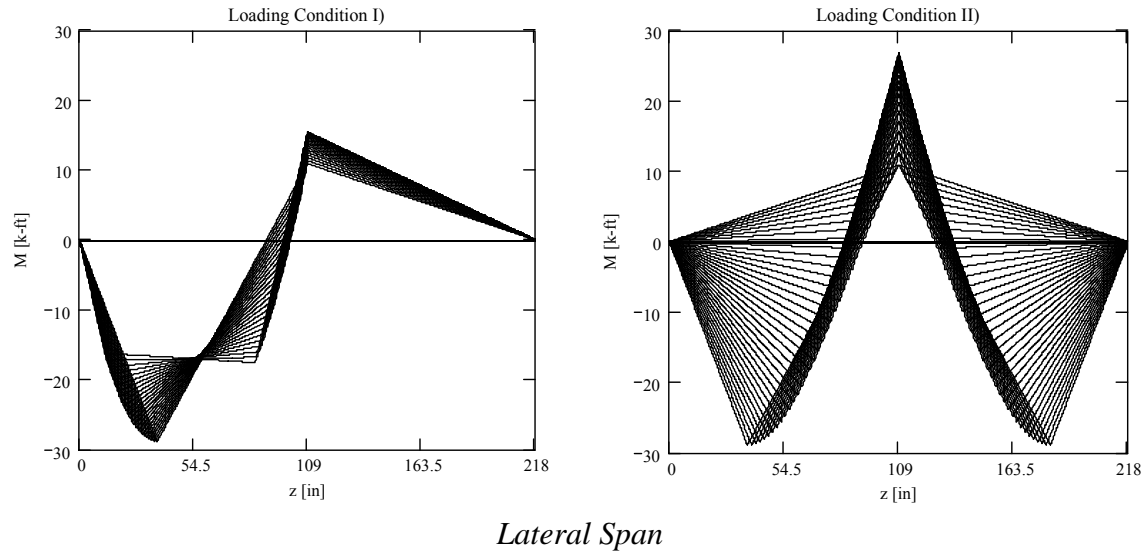


Figure 8 – Live Load Bending Moment Diagram Envelopes per Design Width

## B.4 Transversal Load Distribution to Girders

### B.4.1 Model for Computing Distribution

According to the truck load arrangement of Figure 4, the transversal load distribution can be found by analyzing the structures in Figure 9, where a generic axle of unit weight has been assumed. Because of geometrical restraints, the truck can not drive on the overhang; hence, the small cantilever portion of the deck has been neglected.

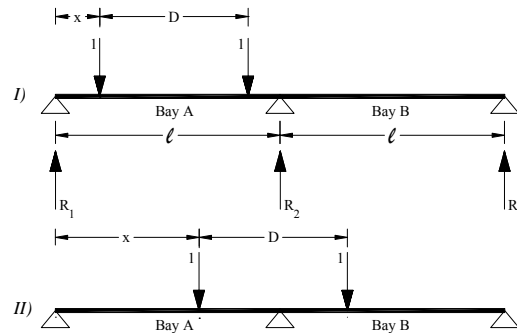


Figure 9 – Transversal Load Distribution: Design Truck Analysis

By increasing the value of  $x$  represented in Figure 9, the design truck load moves from the left to the right portion of the bridge deck. As this movement is allowed, two possible different loading configurations can be recognized as shown in Figure 9I) and II).



The difference between these configurations is related to the number of wheels per bay, as summarized in Table 3. Any other loading condition can be represented by referring to one of the two aforementioned conditions. Table 3 summarizes the values obtained from Figure 9 for the bridge under examination.

Table 3 – Loading Conditions and Bridge Dimensions

Span	Loading Condition	Reference	Bay A	Bay B	$x$ (in)	$\ell$ (in)	$D$ (in)	$d$ (in)
Central	I)	Figure 9-I)	2 wheels	0 wheel	$12 \leq x \leq 35$	107	72	48
	II)	Figure 9-II)	1 wheel	1 wheel	$35 \leq x \leq 107$			
Lateral	I)	Figure 9-I)	2 wheels	0 wheel	$12 \leq x \leq 37$	109		
	II)	Figure 9-II)	1 wheel	1 wheel	$37 \leq x \leq 109$			

A complete analysis of the conditions represented in Figure 9 is carried out according to the protocol of APPENDIX A. In the following, only the results of this analysis are presented.

### B.4.2 Results of the Analysis

Figure 10 shows the load lane conditions when two and one design lane are considered. The calculations related to this analysis are summarized in APPENDIX B.

Figure 11 shows each reaction  $R_1 \dots R_3$  of Figure 9 (which represents the load carried by each girder) as a function of  $x$  for both central and lateral span, respectively.

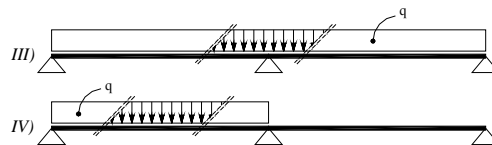
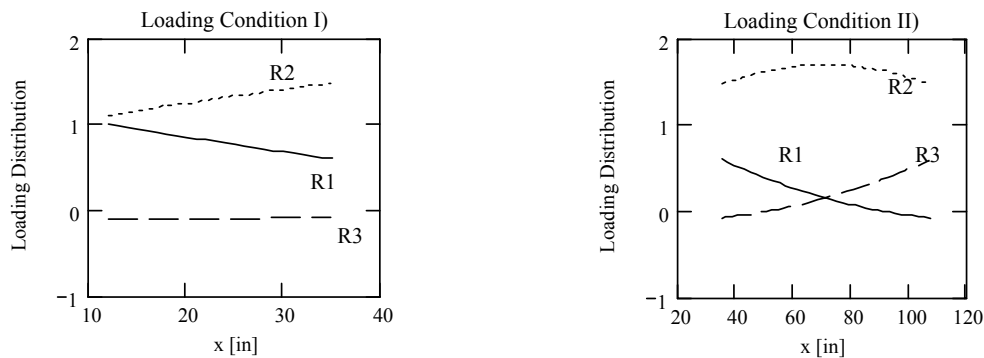


Figure 10 – Transversal Load Distribution: Load Lane Analysis



Central Span

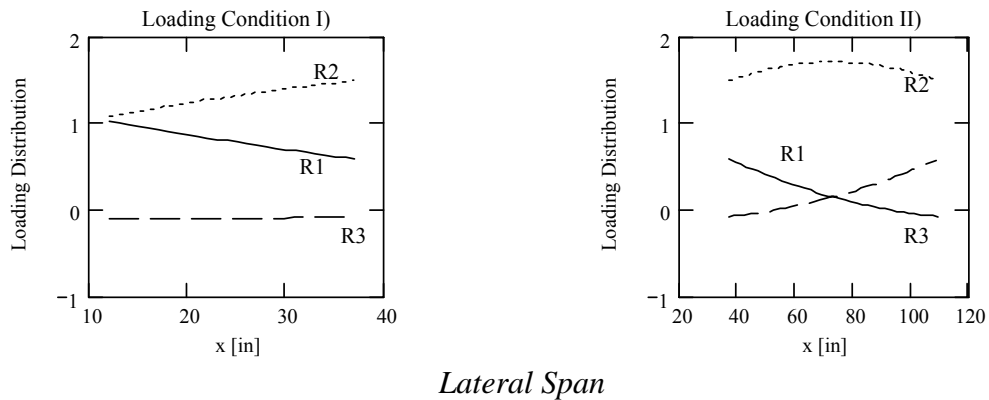


Figure 11 – Reactions as a Function of  $x$

Table 4 summarizes the findings of the distribution of the load to the girders. The  $k_L$  coefficient represents the multiplier of the load to be used in the girder analysis.

Table 4 – Vertical Reactions;  $k_L$  Coefficient

Coefficient	Span	Loading Condition	Exterior Girders $R_1=R_3$	Interior Girder $R_2$
$k_L$	Central	I)	1.00 <sup>a)</sup>	1.473
		II)	0.60	1.698 <sup>a)</sup>
		III)	0.375	1.25 <sup>b)</sup>
		IV)	0.438 <sup>b)</sup>	0.625
	Lateral	I)	1.014 <sup>a)</sup>	1.490
		II)	0.585	1.709 <sup>a)</sup>
		III)	0.375	1.25 <sup>b)</sup>
		IV)	0.438 <sup>b)</sup>	0.625

a) Design values to be used for the design truck analysis of the girders

b) Design values to be used for the load lane analysis of the girders

## B.5 Girder Analysis

### B.5.1 Model for Computing Internal Forces and End Reactions

The analysis is conducted for the three loading conditions recognized in Figure 5 namely: 1) HS20-44 truck load, 2) 3S2 truck load, and 3) load lane.

#### B.5.1.1 Load Lane Analysis

In the last loading condition of Figure 5c), a uniform load of  $0.64 \text{ kip/ft}$  is distributed over the entire length of the girder. Transversely, it is assumed to be uniformly distributed over a  $10 \text{ ft}$  width. Hence, the portion of the uniformly distributed load,  $q$ , carried by each of the three girders can be expressed as follows:

$$q = (0.64)k_L \quad (4)$$

where  $k_L$  represents the fraction of the total load carried by each individual girder. The value of  $k_L$  for interior and exterior girders and for central and lateral span is reported in Table 4.

#### B.5.1.2 HS20-44 and 3S2 Analysis

Figure 12 shows a generic girder with a generic truck load moving on it as the value of  $x_I$  increases from 0 to  $L$ .

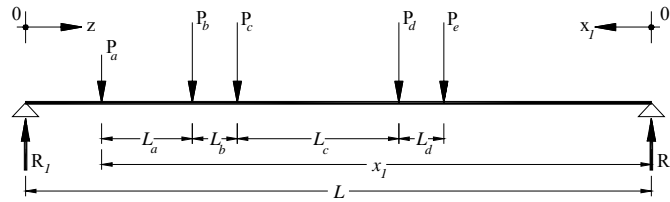


Figure 12 – Design Truck on the Girder

The values of  $P_i$  ( $i=a,b,c,d,e$ ) can be expressed as follows:

$$P_i = k_L P_{wi} \quad (5)$$

where  $P_{wi}$  is the wheel load defined by AASHTO (4 and 16 kip, for an HS20-44 truck load, and 4.64 and 8 kip for a 3S2 truck load) and  $k_L$  represents the fraction of the total load carried by each individual girder. The determination of  $k_L$  is presented in APPENDIX A and its value is summarized in Table 4 for both interior and exterior girders and central and lateral span, respectively.

Table 5 summarizes values reported in Figure 12 and Figure 5c) for the girders under examination and for the three loading conditions being considered.

Table 5 – Parameters for Girder Analysis

Analysis Type	$x_I$ (ft)	$L_a$ (ft)	$L_b$ (ft)	$L_c$ (ft)	$L_d$ (ft)	$L$ (ft)	$P_{wa}$ (kip)	$P_{wb}$ (kip)	$P_{wc}$ (kip)	$P_{wd}$ (kip)	$P_{we}$ (kip)
HS20-44	Varies	14.0	14.0	0.0	0.0	$50^a/40^b$	4.0	16.0	16.0	0.0	0.0
3S2	Varies	12.0	3.8	23.4	3.8	$50^a/40^b$	4.64	8.0	8.0	8.0	8.0
Load Lane <sup>c)</sup>	$L/2$	0.0	0.0	0.0	0.0	$50^a/40^b$	$18.0^d$	0.0	0.0	0.0	0.0
	$d^f$	0.0	0.0	0.0	0.0	$50^a/40^b$	$26.0^e$	0.0	0.0	0.0	0.0

Notes: a) Central Span; b) Lateral Spans; c) Related to the concentrated load analysis only; d) For bending moment analysis; e) For shear force analysis; f) Girder effective depth

As the design truck load moves from the right to the left side of the girder, five different loading conditions are recognized, as shown in Figure 13.

A complete structural analysis protocol for an HS20-44 design truck load is carried out in APPENDIX C.

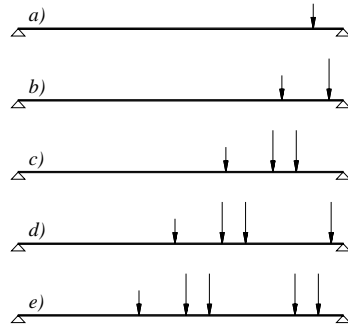


Figure 13 – Design Truck: Possible Loading Conditions

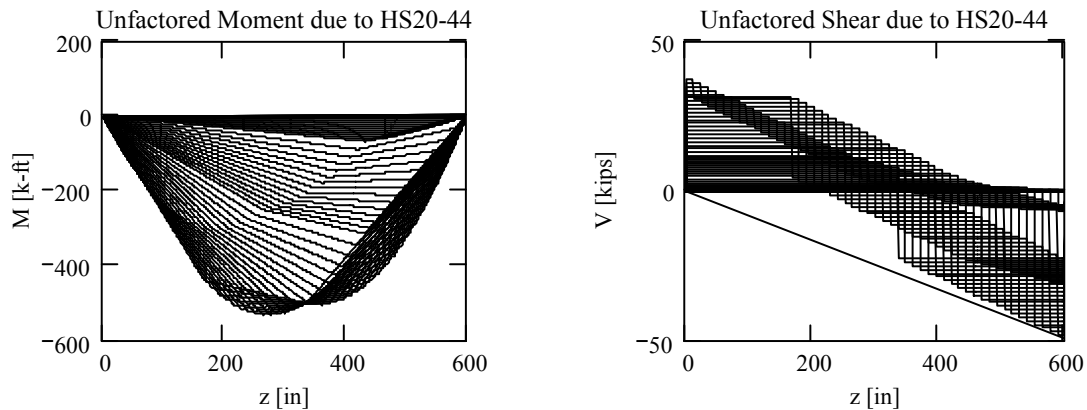
### B.5.2 Results of the Analysis

In the following, only the results needed for the design of critical girder cross-sections are presented. Graphical results will be presented only for the case of the design trucks HS20-44 and 3S2. Results related to the load lane analysis will be summarized later in Table 6 and Table 7.

#### B.5.2.1 Central Span

Figure 14 and Figure 15 show the diagrams of both moments and shear forces as the design trucks move on the interior and exterior girders, respectively.

Bending moment and shear force represented in Figure 14 and Figure 15 refer to both design trucks HS20-44 and 3S2. Ultimate values are then obtained by taking into account the load factors and by adding the moment and shear due to the factored dead load.



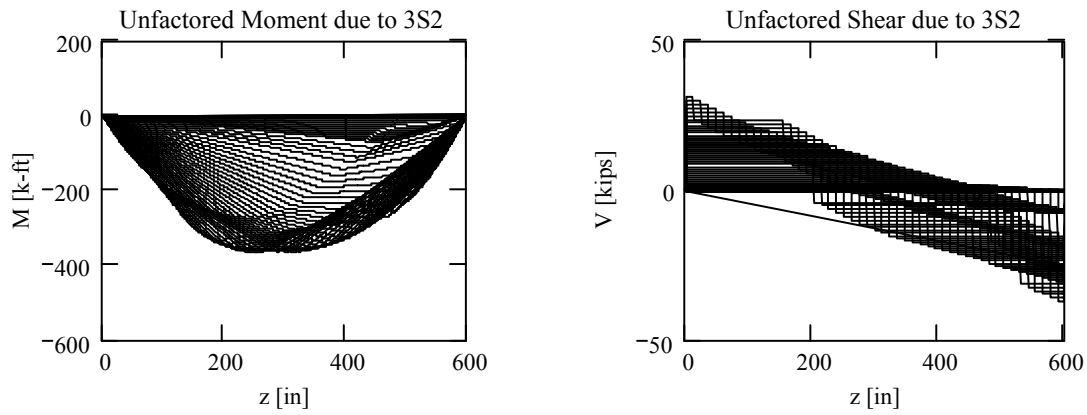


Figure 14 – Live Load Moment and Shear Diagrams for Interior Girders: Central Span

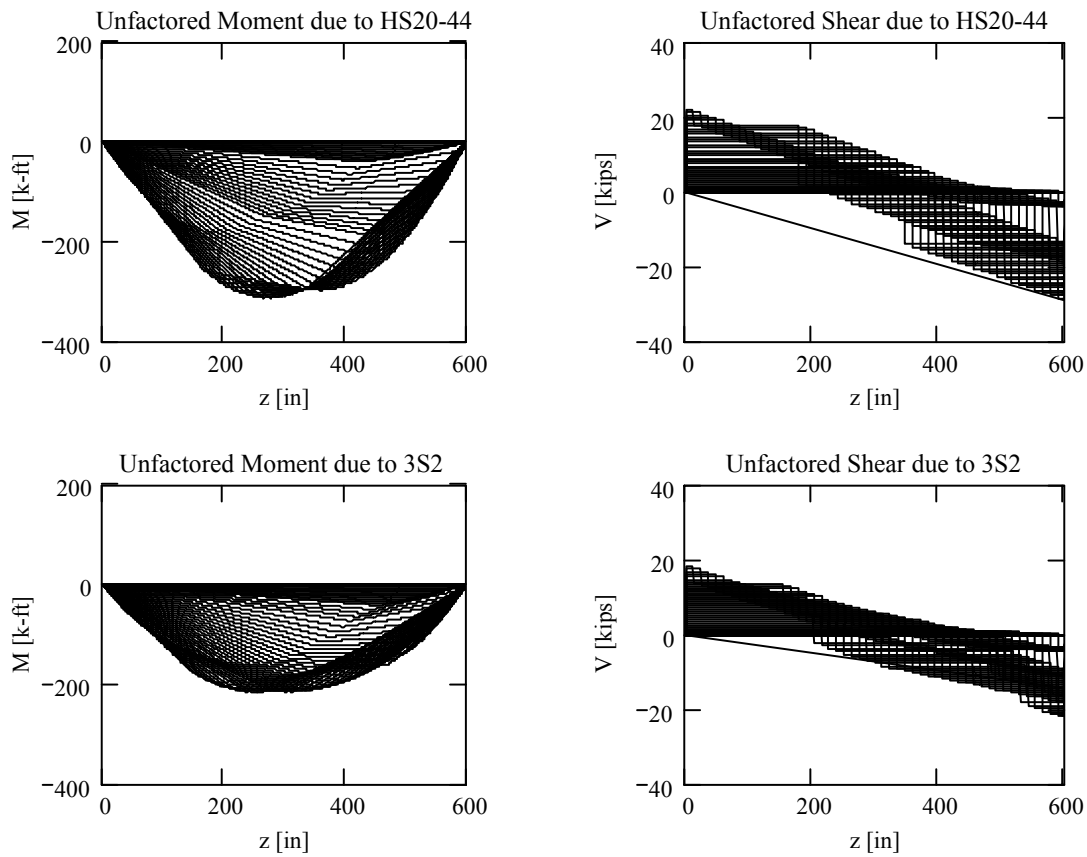


Figure 15 – Live Load Moment and Shear Diagrams for Exterior Girders: Central Span

### B.5.2.2 Lateral Span

Figure 16 and Figure 17 show the diagrams of both moments and shear forces as the design trucks move on the interior and exterior girders, respectively.

Bending moment and shear force represented in Figure 16 and Figure 17 refer to both design trucks HS20-44 and 3S2. Ultimate values are then obtained by taking into account the load factors and by adding the moment and shear due to the dead load.

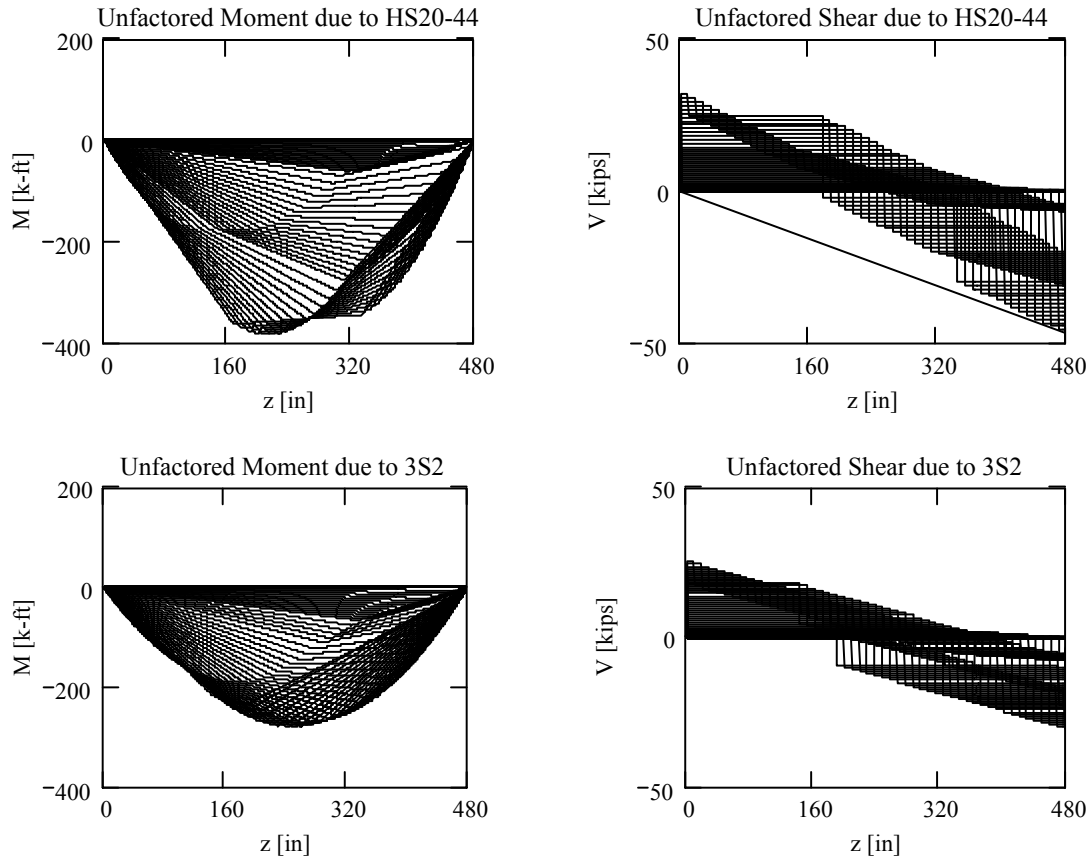


Figure 16 – Live Load Moment and Shear Diagrams for Interior Girders: Lateral Span

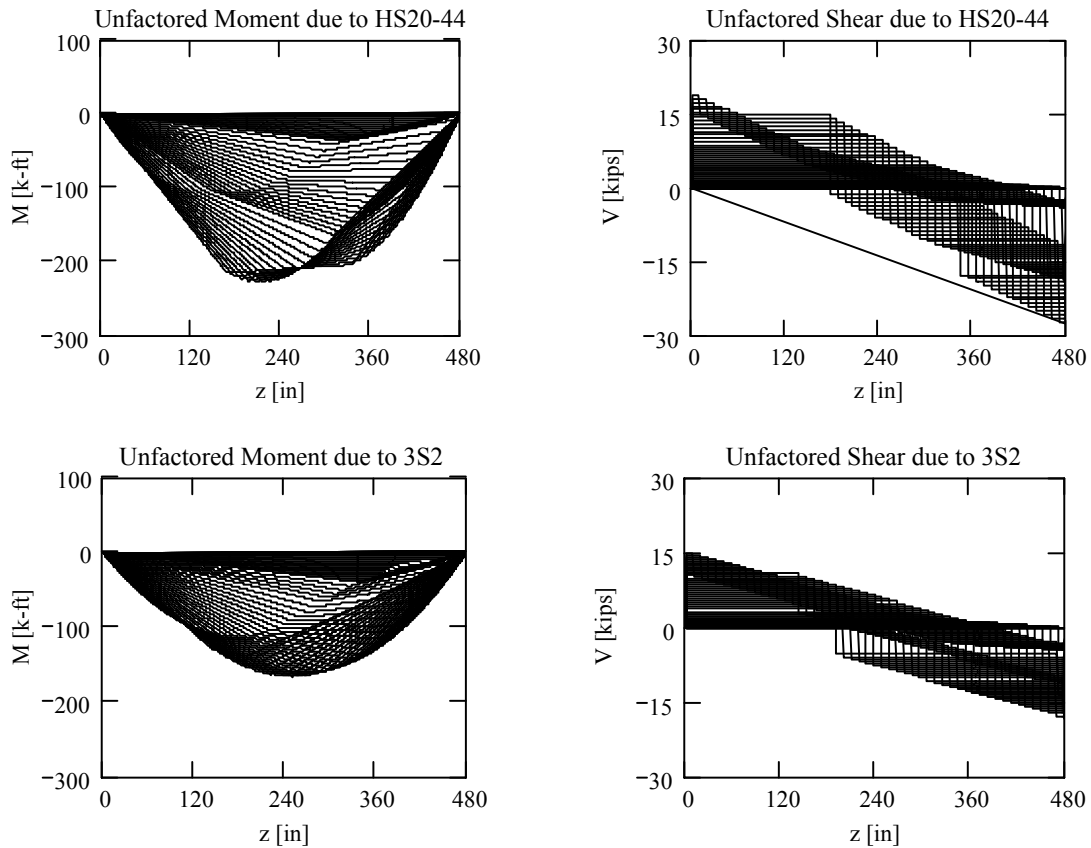


Figure 17 – Live Load Moment and Shear Diagrams for Exterior Girders: Lateral Span

### B.5.3 Load Combinations and Results

Ultimate bending moments and shear forces calculated at several cross-sections, either at a distance  $X$  (for moment) or  $v$  (for shear) from the support (see Figure 18), are summarized in Table 6 and Table 7 for both HS20-44 design truck and load lane. Loading condition 3S2 is not reported because does not control the design as already shown in Figure 14 to Figure 17. The reported values do take into account both factored dead and live load. The cross-sections indicated in Table 6 and Table 7 (i.e., 1-1, A-A etc.) were shown to be critical locations in a preliminary analysis.

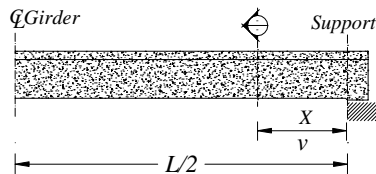


Figure 18 – Identification of Girders Critical Cross-Sections

Table 6 – Breakdown of Moment at Critical Cross-Sections (k-ft)

Span	Girder	Section		Dead Load	Live Load		Factored Load	
		Description	X (in.)		HS20-44	3S2	HS20-44	3S2
Central	Exterior	Support	0	0	0	0	0	0
		1-1	39	165.9	85.8	64.7	456.0	396.9
		2-2	84	328.7	169.7	123.2	902.5	772.3
		3-3	114	420.1	213.2	147.2	1143.2	958.4
		4-4	144	497.9	247.7	172.8	1341.0	1131.2
		5-5	168	550.3	268.8	190.4	1468.2	1248.7
		Mid-span	300	682.4	313.8	216.1	1766.0	1492.4
	Interior	Support	0	0	0	0	0	0
		1-1	48	172.2	179.3	134.5	722.2	600.6
		2-2	96	314.5	319.5	225.5	1296.7	1040.4
		3-3	114	360.2	362.0	250.0	1474.1	1168.3
		4-4	144	426.8	420.6	293.4	1723.7	1376.6
		5-5	168	471.8	456.4	323.2	1881.6	1518.5
		6-6	204	525.1	493.8	349.3	2054.9	1660.9
Mid-span	300	585.0	532.5	367.0	2240.4	1788.4		
Lateral	Exterior	Support	0	0	0	0	0	0
		1-1	27	81.4	58.3	38.5	269.1	213.6
		2-2	60	167.7	117.1	78.4	546.0	437.6
		3-3	114	277.7	183.4	126.6	874.6	715.6
		4-4	144	322.0	204.4	144.3	991.1	822.8
		5-5	168	348.9	215.8	153.6	1057.9	883.7
		6-6	180	359.4	221.6	156.8	1087.8	906.4
	Mid-span	240	383.4	227.9	165.6	1136.6	962.2	
	Interior	Support	0	0	0	0	0	0
		1-1	27	70.1	98.2	64.9	366.1	272.8
		2-2	60	144.4	197.4	132.2	740.5	557.9
		3-3	114	239.0	309.0	213.3	1176.1	908.1
		4-4	144	277.2	344.5	243.1	1325.1	1041.2
		5-5	168	300.3	363.7	259.0	1408.9	1115.7
6-6		204	322.6	383.7	274.5	1493.9	1188.1	
Mid-span	240	330.0	384.2	279.2	1505.0	1210.9		



Table 7 – Breakdown of Shear at Critical Cross-Sections (kip)

Span	Girder	Section		Dead Load	Live Load		Factored Load	
		Description	$v$ (in.)		HS20-44	3S2	HS20-44	3S2
Central	Exterior	Support	0	54.6	29.3	22.5	153.0	134.0
		A-A	18	51.3	27.8	21.3	144.6	126.4
		B-B	36	48.0	27.1	20.5	138.4	119.9
		C-C	84	39.3	24.2	15.7	118.9	95.1
		D-D	102	36.0	23.2	15.0	111.8	88.9
		E-E	200	18.2	17.3	11.7	72.1	56.4
		Mid-span	300	0	10.6	8.0	29.7	22.4
	Interior	Support	0	46.8	49.3	38.2	197.8	167.8
		A-A	18	44.0	47.9	36.1	190.3	158.3
		B-B	36	41.2	46.0	34.8	181.4	151.0
		C-C	72	35.6	42.4	31.1	164.1	133.3
		D-D	84	33.7	41.2	26.6	158.3	118.3
		E-E	200	15.6	29.3	19.8	101.7	75.7
		Mid-span	300	0	18.0	13.5	50.0	37.8
Lateral	Exterior	Support	0	38.3	28.0	18.2	128.3	100.8
		A-A	18	35.5	26.6	17.5	120.6	95.1
		B-B	36	32.6	25.2	16.7	12.9	89.1
		C-C	60	28.8	23.4	15.7	102.9	81.3
		D-D	132	17.3	18.0	12.5	72.8	57.4
		Mid-span	240	0	9.9	7.8	27.7	21.8
	Interior	Support	0	33.0	47.2	30.7	175.1	128.9
		A-A	18	30.5	44.8	29.5	165.1	122.3
		B-B	36	28.0	42.6	28.2	155.8	115.4
		C-C	60	24.7	39.5	26.4	142.8	106.1
		D-D	132	14.8	30.2	21.1	103.9	78.4
		Mid-span	240	0	16.7	13.2	46.8	37.0

## B.6 Bent Analysis

### B.6.1 Model for Computing Vertical Reactions

Bent analysis is carried out considering the structure shown in Figure 19. Vertical loads  $P_1$ ,  $P$  and  $P_2$  are calculated as follows:

$$\begin{aligned}
 P_1 &= (1.3)R_{D,Ext} + (1.3)(1.67)(1.29)R_{L,Ext} \\
 P &= (1.3)R_{D,Int} + (1.3)(1.67)(1.29)R_{L,Int} \\
 P_2 &= (1.3)R_{D,Ext} + (1.3)(1.67)(1.29)R_{L,Ext}
 \end{aligned} \tag{6}$$

where  $R_D$  and  $R_L$  are the vertical reactions due to the dead and live load of girders and deck, respectively, and subscripts *Ext* and *Int* refer to exterior and interior girders.

It is to be noted that the impact factor (1.29) used in Eq. (6) is an average value of the impact factor of the central and lateral spans, respectively (see Table 1).

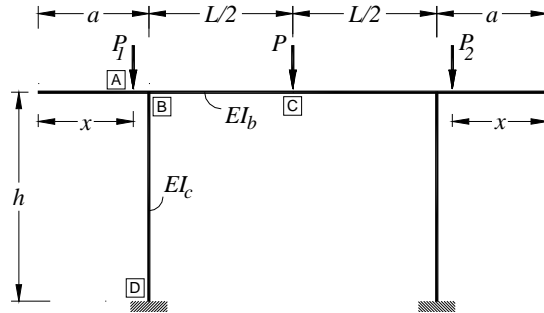


Figure 19 – Bent Frame

Table 8 summarizes the geometrical properties of the frame of Figure 19.

Table 8 – Frame Geometrical Properties

$L$ (in)	$a$ (in)	$x$ (in)	$h$ (in)
198.0	50.0	24.3125	144.0

#### B.6.1.1 Dead Load Analysis

The reactions due to the dead load can be calculated as follows (see Figure 20):

$$R = \frac{\omega_d L}{2} \quad (7)$$

$$\omega_d = h^* W (\gamma_a + \gamma_c) + (2b_1 h_1 + bh) \gamma_c$$

where  $L$  represents the length of the girder,  $h^* = h_a = h_s$  (6 in.), and  $\gamma_a$  and  $\gamma_c$  are the weight per cubic foot of asphalt and concrete, 108 and 150 pcf respectively.

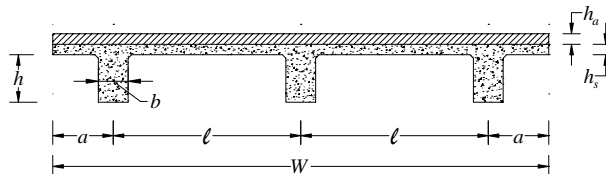


Figure 20 – Determination of the Dead Load of the Superstructure

By neglecting the presence of both overhangs and parapets (conservative assumption), the sharing of the dead load between girders is as reported in Figure 21.

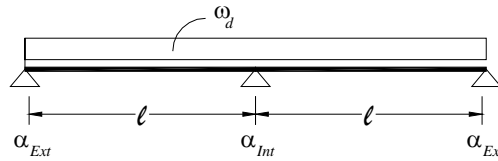


Figure 21 – Load Sharing Between Girders

Table 9 summarizes the findings of the application of Eq. (7) for both central and lateral spans.

Table 9 – Application of Eq. (7)

Span	$W$ (ft)	$L$ (ft)	$\omega_d$ (k/ft)	$R$ (kip)	$R_{tot}$ (kip)
Central	23.6	50.0	5.210	130.3	219.9
Lateral	23.6	40.0	4.479	89.6	

The vertical reactions due to the dead load to be considered for the bent analysis are summarized in Table 10. Their values do consider the sharing distribution of Figure 21 according to the following equation:

$$R_D = \alpha R_{tot} \quad (8)$$

Table 10 – Reactions due to Dead Load

Girder Type	Sharing of Load $\alpha$	Vertical Reaction $R_D$ (kip)
Exterior	0.1875	41.2
Interior	0.625	137.4

#### B.6.1.2 Live Load Analysis: HS20-44

The loading condition imposed on the bent from the superstructure being considered in this analysis is shown in Figure 22. The load of the figure is related to the wheel load of an HS20-44.

Table 11 and Table 12 summarize the results obtained when the loading arrangement on the deck is that of Figure 23. This loading condition corresponds to a truck that moves transversally on the bridge deck.

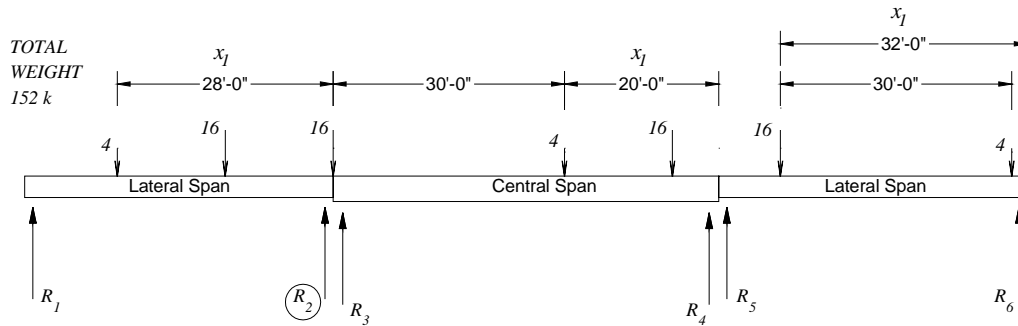


Figure 22 – Bent Loading Condition (Wheel Loads Shown)

Table 11 – Vertical Reactions of the Girders due to an HS20-44

Loading Condition (Figure 23)	Span	$k_L$	Girder Type (Figure 23)	$R_1$ (kip)	$R_2$ (kip)	$R_3$ (kip)	$R_4$ (kip)	$R_5$ (kip)	$R_6$ (kip)
<i>Ia)</i>	Lateral	1.014	Exterior 1	8.5	28.0	0	0	13.2	7.1
		1.092	Interior	9.2	30.1	0	0	14.2	7.6
		-0.105	Exterior 2	-0.9	-2.9	0	0	-1.4	-0.7
	Central	1.000	Exterior 1	0	0	3.5	16.5	0	0
		1.103	Interior	0	0	3.9	18.2	0	0
		-0.103	Exterior 2	0	0	-0.4	-1.7	0	0
<i>Ib)=IIa)</i>	Lateral	0.585	Exterior 1	4.9	16.1	0	0	7.6	4.1
		1.490	Interior	12.5	41.1	0	0	19.4	10.4
		-0.075	Exterior 2	-0.6	-2.1	0	0	-1	-0.5
	Central	0.600	Exterior 1	0	0	2.1	9.9	0	0
		1.473	Interior	0	0	5.2	24.3	0	0
		-0.073	Exterior 2	0	0	-0.3	-1.2	0	0
<i>IIb)</i>	Lateral	0.145	Exterior 1	1.2	4.0	0	0	1.9	1.0
		1.709	Interior	14.4	47.2	0	0	22.2	12
		0.145	Exterior 2	1.2	4.0	0	0	1.9	1.0
	Central	0.151	Exterior 1	0	0	0.5	2.5	0	0
		1.698	Interior	0	0	6.0	28	0	0
		0.151	Exterior 2	0	0	0.5	2.5	0	0

Table 12 – Vertical Loads Acting on the Bent due to an HS20-44

Loading Condition	$R_2+R_3$ (kip)			$R_4+R_5$ (kip)			$R_{tot}$ (kip)
	Exterior 1	Interior	Exterior 2	Exterior 1	Interior	Exterior 2	
<i>Ia)</i>	31.5	34.0	-3.3	29.7	32.4	-3.1	152.0
<i>Ib)=IIa)</i>	18.2	46.3	-2.4	17.5	43.7	-2.2	151.9
<i>IIb)</i>	4.5	53.2	4.5	4.4	50.2	4.4	152.0

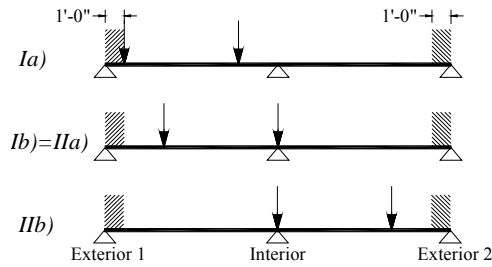


Figure 23 – Transversal Loading Conditions

Table 13 summarizes the values  $R_L$  due to the design truck load to be used in Eq. (6).

Table 13 – Reactions due to HS20-44

Loading Condition	Girder Type	Vertical Reaction $R_L$ (kip)
<i>Ib)=IIa)</i>	Exterior 1	18.2
	Interior	46.3
	Exterior 2	-2.4
<i>IIb)</i>	Exterior 1	4.5
	Interior	53.2
	Exterior 2	4.5

### B.6.1.3 Live Load Analysis: Load Lane

The analysis related to the load lane takes into account both the uniformly distributed load and the concentrated load. For the latter case, only the load of 26 kip of Figure 5c) will be considered in the analysis since it yields to the worst loading condition scenario. The analysis is performed on the structures shown in Figure 24 considering two design lanes.

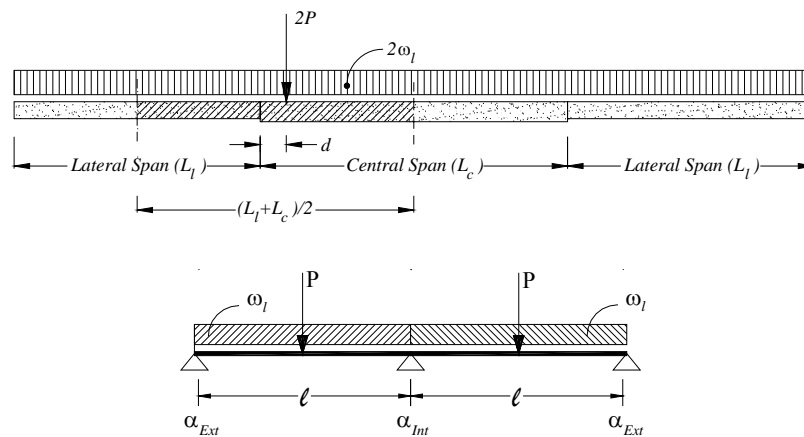


Figure 24 – Loading Condition for the Load Lane Analysis

The vertical reactions  $R_L$  can be expressed as:

$$R_L = \begin{cases} 2\alpha_u \omega_L \frac{L_l + L_c}{2} & \text{uniform load analysis} \\ 2\alpha_c P \frac{L_c - d}{L_c} & \text{concentrated load analysis} \end{cases} \quad (9)$$

where  $\alpha_u$  and  $\alpha_c$  are the sharing of the load between girders (see Table 14),  $\omega_L=0.64$  kip/ft,  $L_l=40$  ft,  $L_c=50$  ft and  $d=33$  in. The results of the application of Eq. (9) are highlighted in Table 14. Table 15 summarizes the load to be used in Eq. (6) when using the load lane analysis.

Table 14 – Application of Eq. (9)

Load Type	Girder Type	Sharing of Load $\alpha_u$ and $\alpha_c$	Vertical Reaction $R_L$ (kip)
Uniform	Exterior	0.1875	10.8
	Interior	0.625	36.0
Concentrated	Exterior	0.15625	7.6
	Interior	0.6875	33.6

Table 15 – Reactions due to Load Lane

Girder Type	Vertical Reaction $R_L$ (kip)
Exterior	10.8+7.6=18.4
Interior	36.0+33.6=69.6

### B.6.2 Vertical Reactions: Results

The application of Eq. (6) gives the results summarized in Table 16 for the two loading conditions namely: 1) HS20-44; and 2) load lane.

Table 16 – Load Transferred to the Bent

Analysis	Loading Condition	$P_1$ (kip)	$P$ (kip)	$P_2$ (kip)
HS20-44	<i>Ib)=IIa)</i>	104.6	308.4	46.9
	<i>IIb)</i>	66.2	327.7	66.2
Load Lane	2 Lanes	105.1	373.6	105.1

### B.6.3 Load Combination and Results

Bending moment and shear force diagrams for the loading condition of Table 16 are shown in Figure 25. A detailed calculation protocol is provided in APPENDIX D. Table 17 summarizes ultimate moments and shear forces calculated at five cross-sections where maximum values are reached.

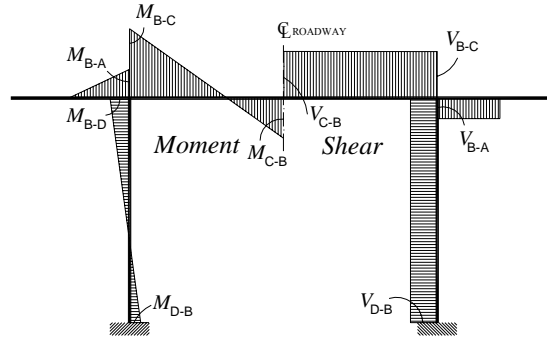


Figure 25 – Bending Moment and Shear Force Diagram

Table 17 – Ultimate Moment and Shear

Analysis	Loading Condition	Section	$M_u$ (k-ft)	$V_u$ (kip)
HS20-44	<i>Ib)=IIa)</i>	B-A	223.9	104.6
		B-C	472.2	159.4
		C-B	843.1	159.4
		B-D	285.4	25.9
		D-B	142.7	25.9
	<i>IIb)</i>	B-A	141.7	66.2
		B-C	442.5	163.8
		C-B	909.3	163.8
		B-D	300.8	27.3
		D-B	150.4	27.3
Load Lane	2 Lanes	B-A	225.0	105.1
		B-C	532.2	186.8
		C-B	1008.9	186.8
		B-D	307.2	27.9
		D-B	153.6	27.9

For non-symmetric loading condition such as *Ib)=IIa)*, values of maximum moment and shear have been considered. They do not necessarily occur at the indicated cross-sections and could be located on the portion of the moment and shear diagram not represented in Figure 25.

In Table 17 and Figure 25 only the results corresponding to the live load are reported. The contribution of the self weight of the member has been neglected.



## C. DESIGN

### C.1 Assumptions

Strengthening design is carried out according to the principles of ACI 440.2R-02 (ACI 440 in the following). The properties of concrete, steel, and FRP laminates used in the design are summarized in Table 18. The reported FRP properties are guaranteed values. The FRP systems used in the design of this bridge are highlighted in Table 18.

The  $\phi$  factors used to convert nominal values to design capacities are obtained as specified in AASHTO for the as-built and from ACI 440 for the strengthened members.

Table 18 – Material Properties

Concrete $f_c$ (psi)	Steel $f_y$ (ksi)	System Type			System Properties		
		NSM System	Manual Lay-up	Pre-cured Laminate	Tensile Strength $f_{fu}^*$ (ksi)	Modulus of Elasticity $E_f$ (ksi) FRP	Size or Thickness $t_f$ (in)
5,450 <sup>a)</sup>	40	Type-1a	-	-	300	19,000	0.079x0.63
		Type-1b	-	-	300	19,000	4/8 bar size
		-	Type-2	-	550	33,000	0.0065
		-	-	Type-3	350	20,000	0.0787

a) From testing of concrete cores

Material properties of the composite reinforcement reported by manufacturers, such as the ultimate tensile strength, typically do not consider long-term exposure to environmental conditions, and should be considered as initial properties. Composite properties to be used in all design equations are given as follows (ACI 440):

$$\begin{aligned}
 f_{fu} &= C_E f_{fu}^* \\
 \varepsilon_{fu} &= C_E \varepsilon_{fu}^*
 \end{aligned}
 \tag{10}$$

where  $f_{fu}$  and  $\varepsilon_{fu}$  are the FRP design tensile strength and ultimate strain considering the environmental reduction factor ( $C_E$ ) as given in Table 8.1 (ACI 440), and  $f_{fu}^*$  and  $\varepsilon_{fu}^*$  represent the FRP guaranteed tensile strength and ultimate strain as reported by the manufacturer (see Table 18). The FRP design modulus of elasticity is the average value as reported by the manufacturer. Properties of the FRP system are related to net fiber area.

## C.2 Slab Design

### C.2.1 Assumptions

Slab geometrical properties and internal steel flexural reinforcement are summarized in Table 19.

Table 19 – Slab Geometrical Properties and Internal Steel Reinforcement

Span	Region	Slab Thickness $h_s$ (in)	Slab Width $b$ (in)	Tensile Steel Area $A_s$ (in <sup>2</sup> /ft)	Effective Depth $d$ (in)	Compression Steel Area $A'_s$ (in <sup>2</sup> /ft)	Effective Depth $d'$ (in)
All Spans	A	6	12	0.96	4.75	0.48	1.75
	B	6	12	0.48	4.75	0.24	1.75

### C.2.2 Positive Moment Strengthening

The strengthening recommendations summarized in Table 20 are suggested for the case of mid-span location (maximum positive moment) for both central and lateral span.

Table 20 – Slab Positive Moment Capacity

FRP Type	Span	Region	Strengthening Scheme	Failure Mode <sup>a)</sup>	$\phi M_n$ (k-ft/ft)	$M_u$ (k-ft/ft)
Type-2	All	A	No FRP	CC	13.8	12.2
		B	No FRP	CC	7.5	
			1 Ply 6" wide @14" ocs	CC	12.4	

a) CC=Concrete Crushing,

Slab flexural strengthening of the positive moment region is shown in Figure 26 and Figure 27.

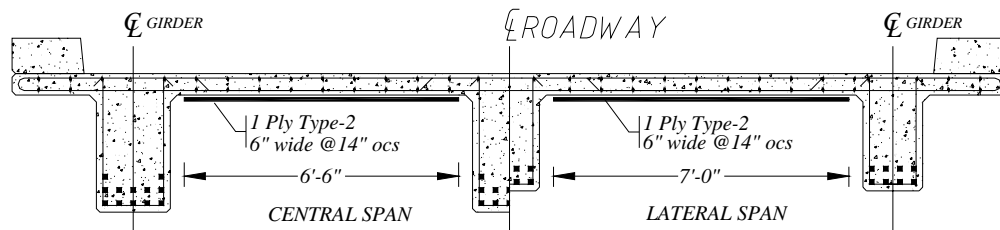


Figure 26 – Slab Cross-Section

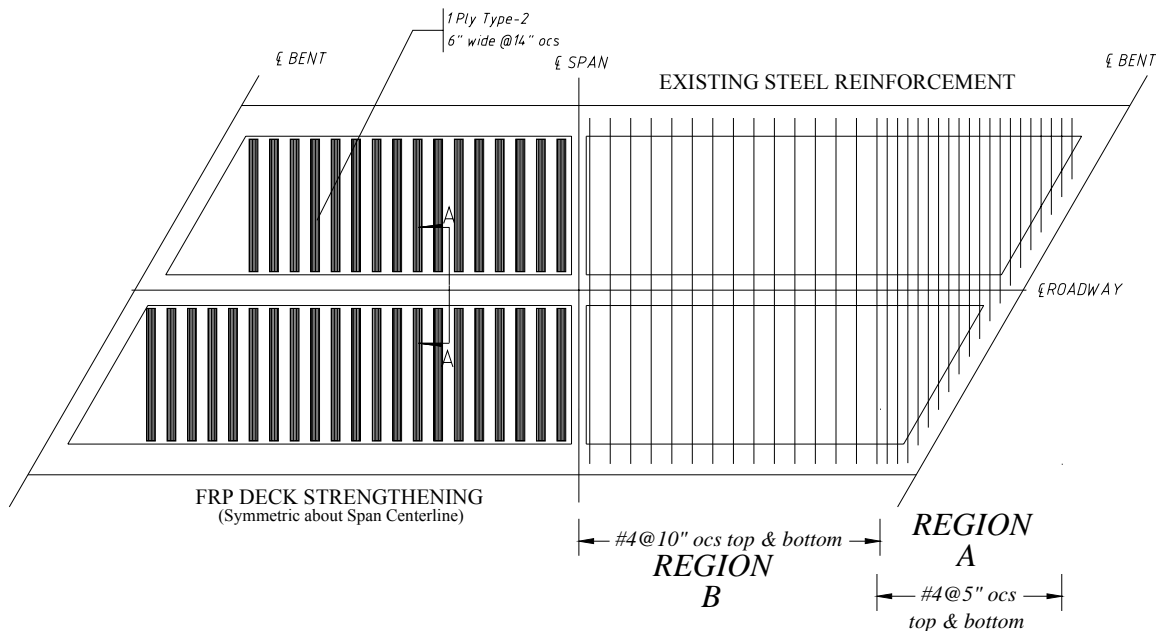


Figure 27 – Plan View of Slab Strengthening

### C.2.3 Negative Moment Check

Strengthening of the negative moment region of the bridge deck is not a viable solution. The as-built negative moment capacity is summarized in Table 21. Both spans do not need any flexural strengthening because their as-built capacity is acceptable.

Table 21 – Slab Negative Moment Capacity

Span	Region	Failure Mode	$\phi M_n$ (k-ft/ft)	$M_u$ (k-ft/ft)
All	A	CC	13.8	8.8
	B	CC	7.5	

CC=Concrete Crushing

Note that both positive and negative moment acting on the deck have been calculated using the center-to-center distance between supports. AASHTO Section 3.24.1.1 allows the use of a smaller span length ( $L_{net} + 2h_s$ ) that further reduces both positive and negative flexural demand.

## C.2.4 Shear Check

Shear strengthening of slab-deck systems is not a viable solution. The as-built shear capacity is summarized in Table 22. No shear strengthening will be provided on the slab since the values of the as built shear capacity and demand are sufficiently close.

Table 22 – Slab Shear Capacity

Span	Region	$\phi V_n$ (kip/ft)	$V_u$ (kip/ft)
Central	A	8.2	8.6
	B	7.5	
Lateral	A	8.6	9.0
	B	7.7	

The concrete shear capacity of the slab has been calculated using the more detailed equation of ACI 318-99<sup>4</sup> (see later Eq. (12)). Ultimate moments at the same cross-section where the shear is evaluated are summarized in Table 23.

Table 23 – Ultimate Moment for Shear Capacity

Span	Loading Condition (See Figure 7)	$M_u$ (k-ft/ft)
Central	I)	5.0
Lateral	I)	4.0

Table 22 reports data for  $V_u$  computed considering the resultant load from each set of two tires. Since each tire has a width of 20 in. (AASHTO Section 3.30), the distributed load would have a total width of  $4 \times 20 = 80$  in. over a span of 109 in. If one were to consider this load uniformly distributed the equivalent load per linear foot would be  $q = 0.56$  kip/ft, which would correspond to an ultimate shear computed at the same location of Table 22 of  $V_u = 7.4$  kip/ft. This value represents a lower bound while the one in Table 22 is the upper bound.

## C.3 Girders Design

### C.3.1 Assumptions

Girder geometrical properties are reported in Table 24, Table 25, Table 26, and Figure 28a), b) and c) summarize internal flexural and shear reinforcement at different cross-sections where there is a change in the lay-out of the reinforcement.

The expression for the flange width,  $b_{eff}$ , is given by the following equations, according to AASHTO Section 8.10.1 for interior and exterior girders, respectively:

$$b_{eff}^{Int} = \min\left(\frac{L}{4}, 12h_s + b, \ell\right)$$

$$b_{eff}^{Ext} = b + \min\left(\frac{L}{12}, 6h_s, \frac{\ell - b}{2}\right)$$
(11)

where  $L$  is the girder length,  $h_s$  and  $b$  are defined in Table 24, and  $\ell$  represents the center-to-center girder spacing.

Table 24 – Geometrical Properties

Span	Girder Type	Overall Height $h$ (in)	Width of the Web $b$ (in)	Width of the Flange $b_{eff}$ (in)	Slab Thickness $h_s$ (in)
Central	Interior	39	21	93	6
	Exterior	39	21	57	6
Lateral	Interior	33	17	89	6
	Exterior	33	17	53	6

Table 25 – Flexural Internal Steel Reinforcement

Span	Girder Type	Section (Figure 28)	Tensile Steel Area $A_s$ (in <sup>2</sup> )	Effective Depth $d$ (in)	Compression Steel Area $A'_s$ (in <sup>2</sup> )	Effective Depth $d'$ (in)
Central	Interior	Support	7.8125	36.5	2.10	1.75
		1-1	12.50	35.1	2.10	1.75
		2-2 to Mid-span	15.625	34.625	2.10	1.75
	Exterior	Support	7.8125	36.5	1.56	1.75
		1-1	12.50	35.1	1.56	1.75
		2-2 to Mid-span	15.625	34.625	1.56	1.75
Lateral	Interior	Support	6.25	30.5	2.01	1.75
		1-1	9.375	29.25	2.01	1.75
		2-2 to Mid-span	12.50	28.625	2.01	1.75
	Exterior	Support	6.25	30.5	1.47	1.75
		1-1	9.375	29.25	1.47	1.75
		2-2 to Mid-span	12.50	28.625	1.47	1.75

Table 26 – Shear Internal Steel Reinforcement

Span	Girder Type	Section (see Figure 28)	Stirrup Area $A_{vs}$ (in <sup>2</sup> )	Stirrups Spacing $s_s$ (in)	Bent Bar Area $A_{vb}$ (in <sup>2</sup> )	Bent Bar Spacing $s_b$ (in <sup>2</sup> )
Central	Interior/ Exterior	Support	0.40	6	4.6875	33
		A-A	0.40	9	4.6875	33
		B-B	0.40	12	3.125	33 Int/27.5 Ext
		C-C	0.40	12	0 Int/3.125 Ext	0 Int/33 Ext
		D-D to Mid-span	0.40	18	0	0
Lateral	Interior/ Exterior	Support to B-B	0.40	12	3.125	28.5
		C-C to Mid-span	0.40	18	0	0

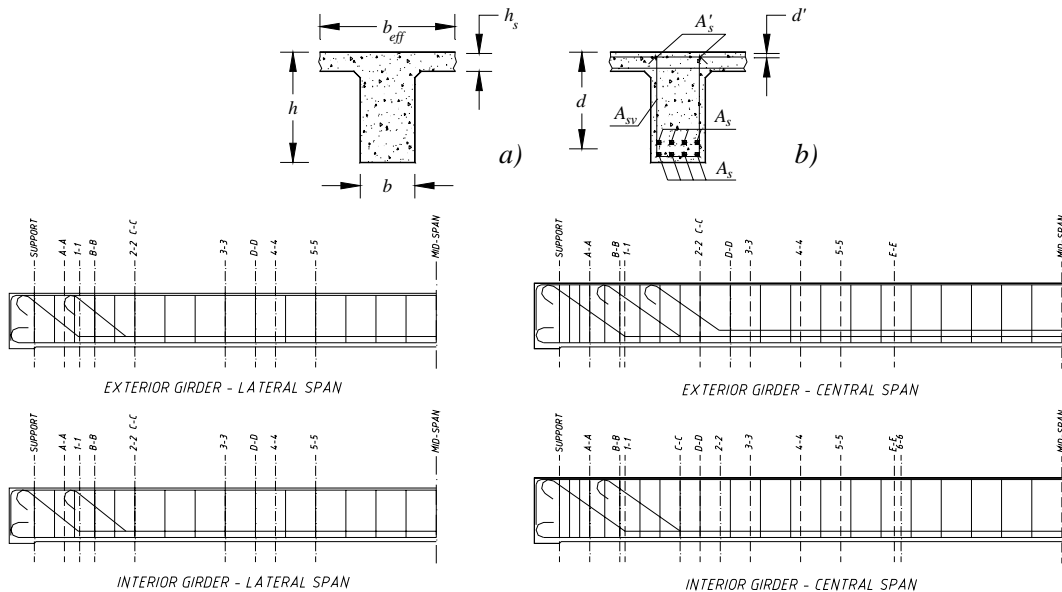


Figure 28 – Girder Dimensions and Internal Reinforcement

### C.3.2 Positive Moment Strengthening

Table 27 summarizes the achieved flexural capacity at mid-span for interior and exterior girders as a function of the adopted strengthening scheme.

When FRP laminates are used, the bond dependent coefficient,  $\kappa_m$ , defined by Eq. (9-2) of ACI 440, accounts for cover delamination or FRP debonding that could occur if the force in the FRP cannot be sustained by the substrate. When FRP U-Wraps are installed to anchor the external flexural reinforcement, the value of  $\kappa_m$  can be taken up to 0.9 since both cover delamination and FRP debonding are effectively prevented.

Table 27 – Girders Flexural Capacity at Mid-span

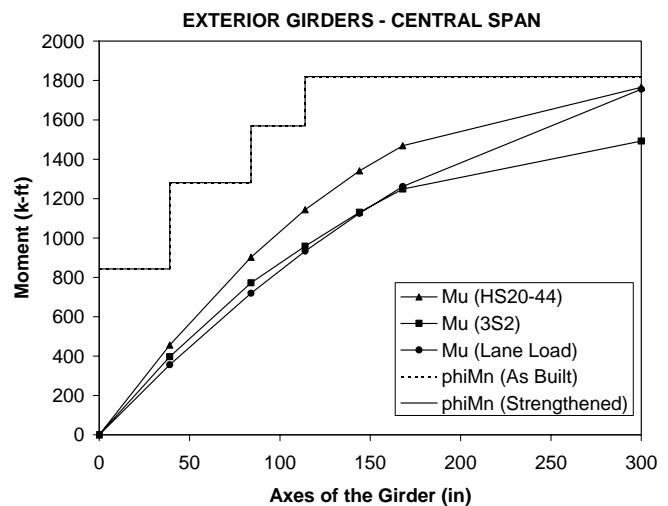
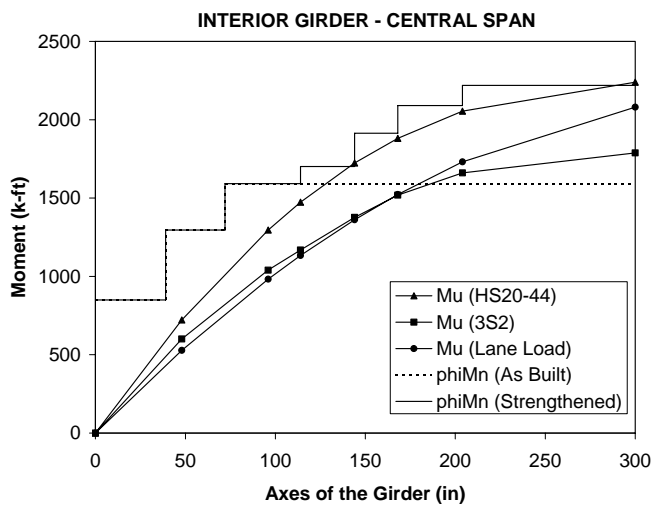
FRP Type	Span	Girder Type	Description	Failure Mode <sup>a)</sup>	$\kappa_m$ (-)	$\phi M_n$ (k-ft)	$M_u$ (k-ft)
Type-2	Central	Interior	No FRP	CC	-	1591.9	2240.4
			5 Plies 20" wide	TC	0.9	2220.0	
		Exterior	No FRP	CC	-	1820.0	1766.0
			Lateral	Interior	No FRP	CC	-
	5 Plies 16" wide + 1 Lateral #4 NSM Bar each side	TC			0.9	1519.7	
	Exterior	No FRP	CC	-	1035.9	1164.5 <sup>b)</sup>	
		2 Plies 16" wide	TC	0.9	1200.9		

a) CC=Concrete Crushing; TC=Tension Control

b) Design Lane

When adding FRP, the failure mode is usually governed by FRP rupture because of its limited ultimate strain at failure as compared to that of steel. This also represents an optimal use of an expensive material. Only when the amount of applied FRP becomes larger, the failure mode changes from tension controlled (FRP rupture) to concrete crushing.

Figure 29 shows the flexural demand and the as built and strengthened capacities of both central and lateral spans, and interior and exterior girders, respectively. The demand has been shown for the three loading conditions analyzed.



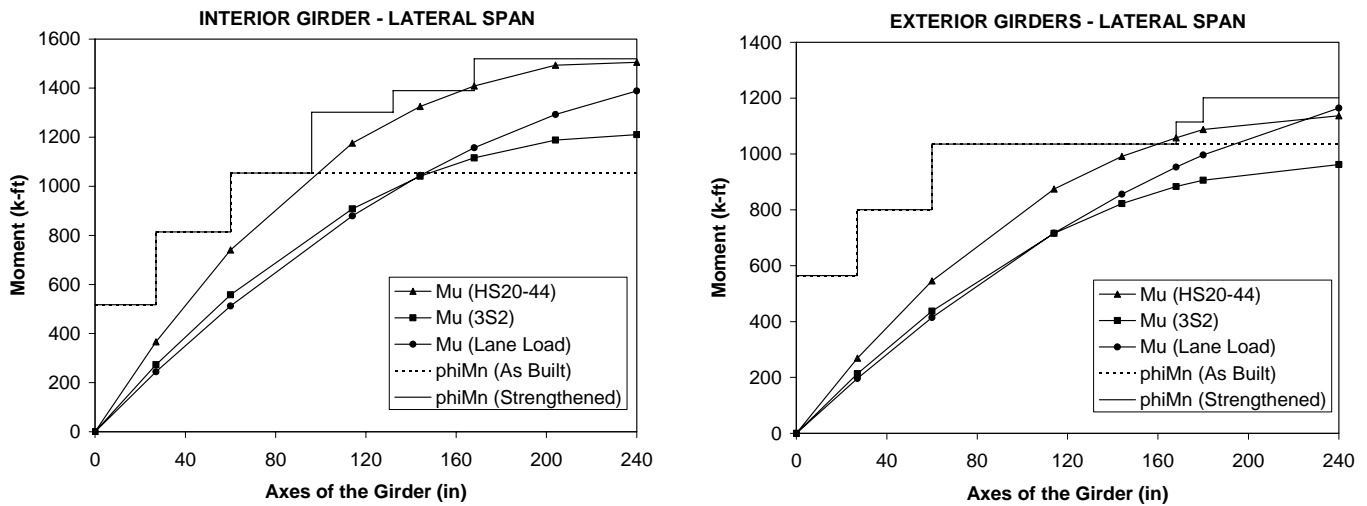


Figure 29 – Flexural Demand and Flexural Capacity

A sketch showing the layout of FRP flexural reinforcement is presented in Figure 30. As the ultimate moment decreases towards the supports, the total number of plates reduces. Since FRP sheets cannot be used on the side faces of the girders due to the presence of the transverse beam, NSM bars are used as shown in Figure 30.

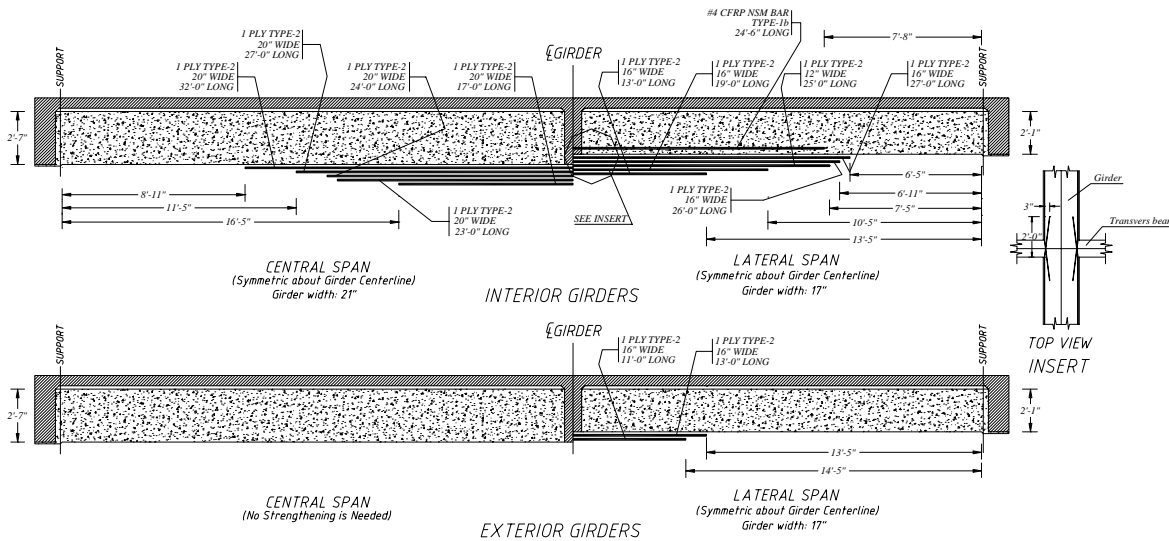


Figure 30 – FRP Flexural Strengthening

### C.3.3 Negative Moment Check

All girders are simply supported, and therefore no negative moment exists.



### C.3.4 Shear Strengthening

The concrete contribution to the shear capacity has been assumed to be based on Eq. (11-5) of ACI 318-99 as follows:

$$V_c = \left( 1.9\sqrt{f'_c} + 2500\rho_w \frac{V_u d}{M_u} \right) b_w d \leq 3.5\sqrt{f'_c} b_w d \quad (12)$$

where  $\rho_w = A_w/b_w d$ ,  $V_u$  and  $M_u$  represent ultimate bending moment and shear force acting at the same cross-section, respectively, and  $b_w$  and  $d$  are width and effective depth of the girder. The steel contribution to the shear capacity can be expressed as follows:

$$V_s = \frac{A_{vs} f_y d}{s_s} + \frac{A_{vb} f_y (\sin \alpha + \cos \alpha) d}{s_b} \leq 2 \frac{A_{vs} f_y d}{s_s} \quad (13)$$

where  $\alpha$  represents the slope of the bent bar ( $\alpha=45^\circ$ ), and all other symbols are indicated in Table 26. The limitation expressed by the last term of Eq. (13) is a conservative assumption to take into account the localized effect exerted by the bent bars.

The FRP contribution to the shear capacity is expressed as follows (ACI 440):

$$V_f = \frac{A_{fv} f_{fe} d_f}{s_f} \quad (14)$$

where  $A_{fv}$  is the FRP laminate area,  $f_{fe}$  is the effective tensile strength allowable to the FRP reinforcement,  $d_f$  is the depth of the FRP reinforcement, and  $s_f$  is the FRP spacing.

Table 28 summarizes the achieved shear capacity close to quarter-span where most of the FRP strengthening is necessary for both the central and lateral spans, and interior and exterior girders as a function of the adopted strengthening scheme.

Table 28 – Girders Shear Capacity close to Quarter-Span

FRP Type	Span	Girder Type	Section	Description	$\kappa_v$ (-)	$\phi V_n$ (kip)	$V_u$ (kip)
Type-2	Central	Interior	D-D	No FRP	-	123.0	158.3
				1 Ply Continuous, U-wrap	0.346	160.4	
	Lateral	Interior	C-C	No FRP	-	89.6	142.8
				2 Plies Continuous, U-wrap	0.234	134.3	
		Exterior	C-C	No FRP	-	88.5	102.9
				1 Ply 12" wide @24" ocs, U-wrap	0.337	104.4	

Figure 31 shows the as built and strengthened shear capacities compared to the shear demand for all girders. A sketch showing the layout of FRP shear reinforcement is presented in Figure 32.

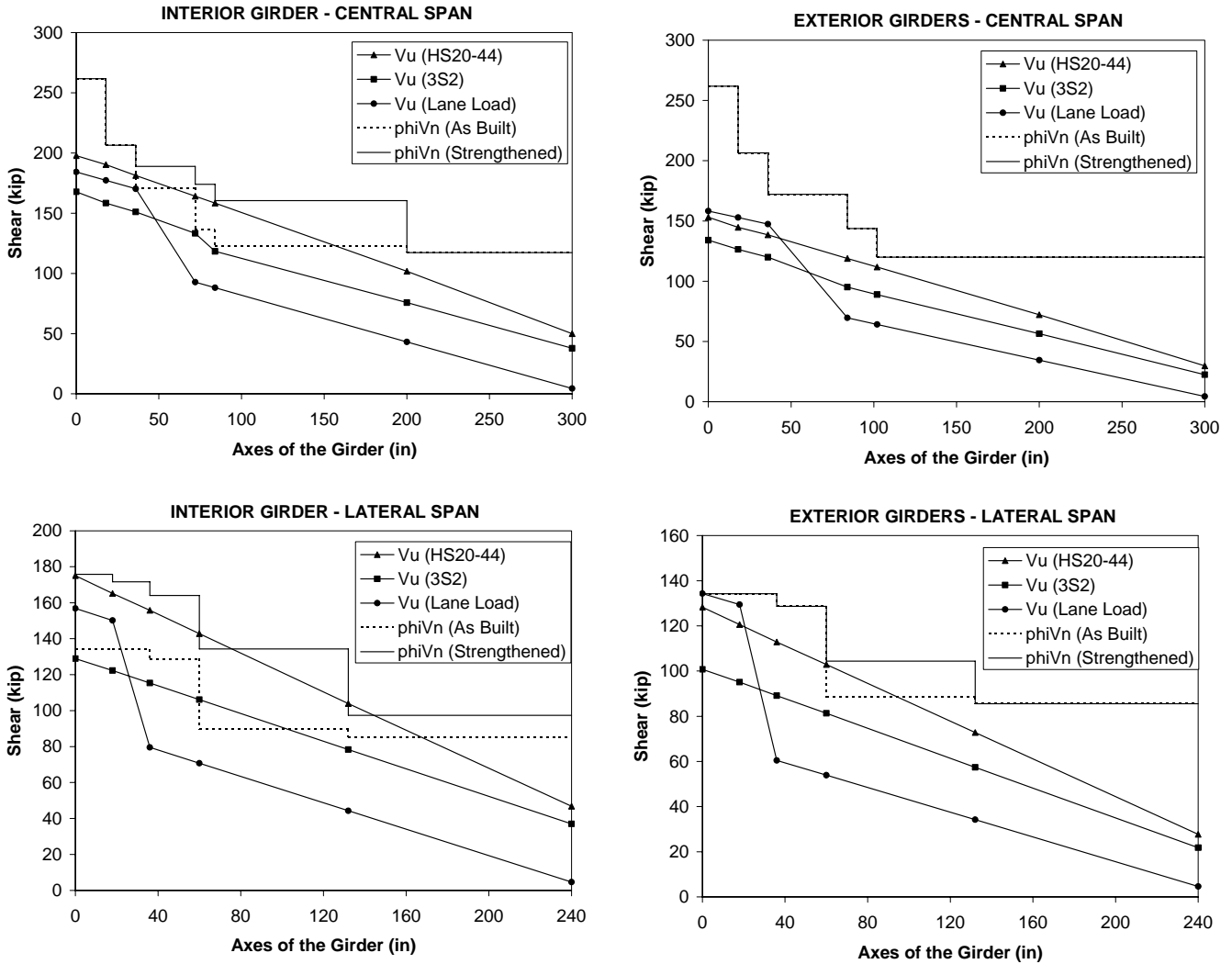


Figure 31 – Shear Demand and Shear Capacity

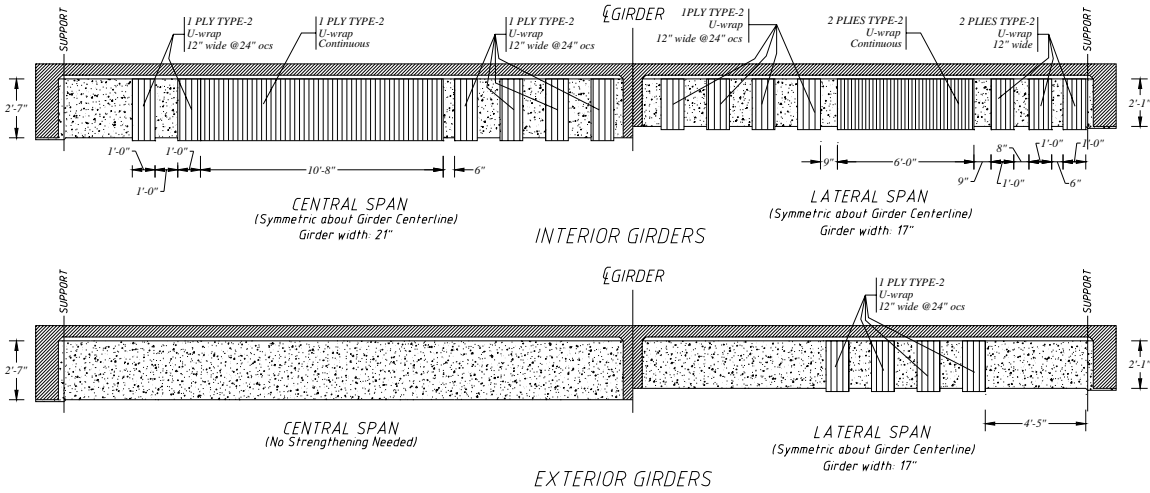


Figure 32 – FRP Shear Strengthening

## C.4 Bent Design

### C.4.1 Assumptions

Bent geometrical properties are summarized in Figure 33 and Table 29.

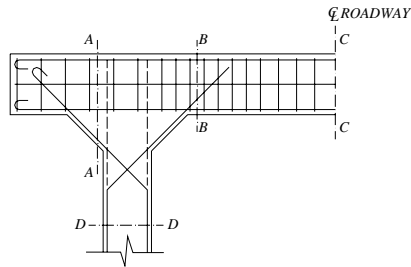


Figure 33 – Portion of the Bent

Table 29 – Bent Geometrical Properties

Member Type	Section	Overall Height $h$ (in)	Width $b$ (in)	Steel Area $A_s$ (in <sup>2</sup> )	Effective Depth $d$ (in)	Steel Area $A'_s$ (in <sup>2</sup> )	Effective Depth $d'$ (in)	Stirrups Area $A_v$ (in <sup>2</sup> )	Stirrups Spacing $s$ (in)
Beam	A-A	48	42	6.0	27.5	6.0	3.0	0.40	12
	B-B	30	42	6.0	27.5	6.0	3.0	0.40	12
	C-C	30	42	6.0	27.5	6.0	3.0	0.40	12
Pier	D-D	27	27	1.32	24.5	1.32	24.5	0.22	12

### C.4.2 Positive Moment Strengthening

The beam flexural capacity is summarized in Table 30. Flexural strengthening is needed for the positive moment region of the beam of the bent cup.

Table 30 – Positive Moment Flexural Capacity

FRP Type	Section (Figure 33)	Strengthening Description	$\kappa_m$ (-)	$\phi M_n$ (k-ft)	$M_u$ (k-ft)
Type-2	C-C	No FRP	-	497.5	1008.9
		3 Plies 42" wide	0.796	1020.4	

### C.4.3 Negative Moment Check

The beam does not need strengthening in its negative moment region. The as-built flexural capacity and flexural demand are summarized in Table 31 for the cross-section of interest.

Table 31 – Negative Moment Flexural Capacity

Section (Figure 33)	Strengthening Description	$\phi M_n$ (k-ft)	$M_u$ (k-ft)
A-A	No FRP	497.5	532.2

Note that the negative moment check has been done using the ultimate moment of cross-section A-A and the geometrical properties and steel reinforcement lay-out of cross-section B-B of Figure 33. The as-built negative moment capacity can be considered acceptable.

### C.4.4 Shear Capacity Check

Shear strengthening of the bent cup is not required as shown in Table 32.

Table 32 – Bent Cup Shear Capacity

Section (Figure 33)	Strengthening Description	$\phi V_n$ (kip)	$V_u$ (kip)
A-A	No FRP	176.1	186.8

Once again, the shear check has been done assuming the ultimate shear of cross-section A-A and the geometrical properties and steel reinforcement lay-out of cross-section B-B of Figure 33. The as-built shear capacity can be considered acceptable.

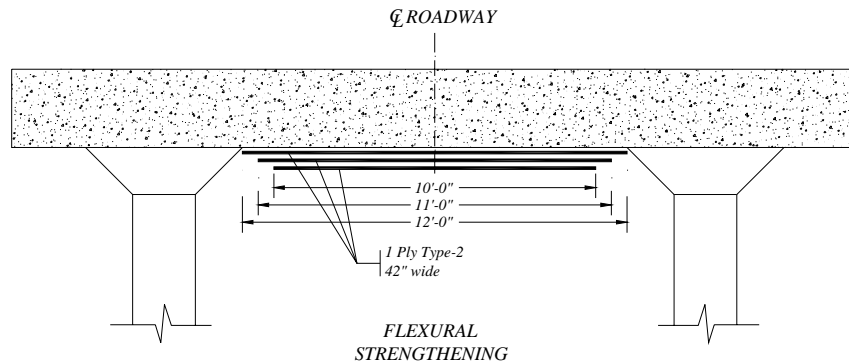


Figure 34 – Strengthening of the Bent

#### C.4.5 Piers Check

Flexural and axial load capacities of the piers need not be upgraded because the ultimate moment and axial load demand is inside the P-M diagram of the members as shown in Figure 35. Values of ultimate axial load and bending moment are  $P_u=291.9 \text{ kip}$  and  $M_u=307.2 \text{ k-ft}$ , respectively.

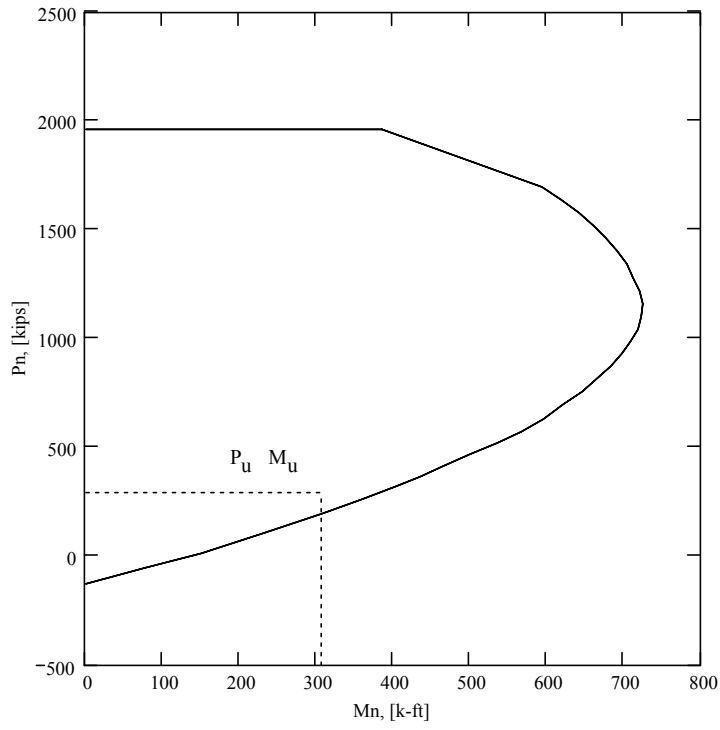


Figure 35 – Pier Flexural and Axial Load Capacity

## D. LOAD RATING

Bridge load rating calculations provide a basis for determining the safe load carrying capacity of a bridge. According to MoDOT, anytime a bridge is built, rehabilitated, or reevaluated for any reason, inventory and operating ratings are required using the Load Factor rating method. All bridges should be rated at two load levels, the maximum load level called the Operating Rating and a lower load level called the Inventory Rating. The Operating Rating is the maximum permissible load that should be allowed on the bridge. Exceeding this level could damage the bridge. The Inventory Rating is the load level the bridge can carry on a daily basis without damaging the bridge.

In Missouri, for the Load Factor Method the Operating Rating is based on the appropriate ultimate capacity using current AASHTO specifications (AASHTO, 1994). The Inventory Rating is taken as 86% of the Operating Rating.

The vehicle used for the live load calculations in the Load Factor Method is the HS20 truck. If the stress levels produced by this vehicle configuration are exceeded, load posting may be required.

The method for determining the rating factor is that outlined by AASHTO in the Manual for Condition Evaluation of Bridges (AASHTO, 1996). Equation (15) was used:

$$RF = \frac{C - A_1 D}{A_2 L (1 + I)} \quad (15)$$

where: RF is the Rating Factor, C is the capacity of the member, D is the dead load effect on the member, L is the live load effect on the member, I is the impact factor to be used with the live load effect,  $A_1$  is the factor for dead loads, and  $A_2$  is the factor for live loads. Since the load factor method is being used,  $A_1$  is taken as 1.3 and  $A_2$  varies depending on the desired rating level. For Inventory rating,  $A_2 = 2.17$ , and for Operating Rating,  $A_2 = 1.3$ .

To determine the rating (RT) of the bridge Equation (16) was used:

$$RT = (RF)W \quad (16)$$

In the above equation, W is the weight of the nominal truck used to determine the live load effect.

For Bridge X-0495, the Load Rating was calculated for a number of different trucks, HS20, H20, 3S2, and MO5. The different ratings are used for different purposes by the bridge owner. For each of the different loading conditions, the maximum shear and maximum moment were calculated. An impact factor is also taken into account for load rating. This value for Bridge X-0495 is 28% and 30% for the central and lateral span respectively. The live load effect of each truck on the different elements of the bridge was determined using the same methodology already described in the APPENDICES A-D.

## D.1 Slab Rating

The shear and positive and negative moment values for the slab are shown below in Table 33 and Table 34 for the central and lateral spans respectively.

Table 33 - Maximum Shear and Positive and Negative Moments due to Live Loads for the Central Span Slabs

Truck	Maximum Positive Moment [kip-ft/ft]	Maximum Negative Moment [kip-ft/ft]	Maximum Shear [kip/ft]	Maximum Positive Moment with Impact Factor [kip-ft/ft]	Maximum Negative Moment with Impact Factor [kip-ft/ft]	Maximum Shear with Impact Factor [kip/ft]
HS20	3.9	2.1	2.9	5.0	2.6	3.8
MO5	2.0	1.0	2.4	2.5	1.3	3.0
H20	3.4	1.5	2.4	4.4	1.9	3.0
3S2	3.4	1.5	2.4	4.4	1.9	3.0

Table 34 - Maximum Shear and Positive and Negative Moments due to Live Loads for the Lateral Spans Slabs

Truck	Maximum Positive Moment [kip-ft/ft]	Maximum Negative Moment [kip-ft/ft]	Maximum Shear [kip/ft]	Maximum Positive Moment with Impact Factor [kip-ft/ft]	Maximum Negative Moment with Impact Factor [kip-ft/ft]	Maximum Shear with Impact Factor [kip/ft]
HS20	3.9	2.1	2.9	5.1	2.7	3.8
MO5	2.0	1.0	2.4	2.6	1.3	3.1
H20	3.4	1.5	2.4	4.5	2.0	3.1
3S2	3.4	1.5	2.4	4.5	2.0	3.1

Table 35 and Table 36 give the results of the Load Rating pertaining to positive and negative moments respectively for the central span, while Table 37 shows the results for the shear forces for the same span.



Table 35 - Rating Factor for the Central Span Slab (Positive Bending Moments)

Truck	Rating Factor (RF)	Rating (RT) (Tons)	Rating Type
HS20	1.731	62.3	Operating
HS20	1.037	37.3	Inventory
MO5	3.462	124.6	Operating
H20	1.708	34.2	Posting
3S2	1.708	62.6	Posting

\* All Units Expressed in English System

Table 36 - Rating Factor for the Central Span Slab (Negative Bending Moments)

Truck	Rating Factor (RF)	Rating (RT) (Tons)	Rating Type
HS20	3.478	125.2	Operating
HS20	2.084	75.0	Inventory
MO5	6.956	250.4	Operating
H20	4.110	82.2	Posting
3S2	4.110	150.6	Posting

\* All Units Expressed in English System

Table 37 - Rating Factor for the Central Span Slab (Shear)

Truck	Rating Factor (RF)	Rating (RT) (Tons)	Rating Type
HS20	1.465	52.7	Operating
HS20	0.878	31.6	Inventory
MO5	1.824	66.8	Operating
H20	1.569	31.4	Posting
3S2	1.569	57.5	Posting

\* All Units Expressed in English System

Table 38 and Table 39 give the results of the Load Rating pertaining to positive and negative moments respectively for the lateral spans, while Table 40 shows the results for the shear forces for the same span.

Table 38 - Rating Factor for the Central Span Slab (Positive Bending Moments)

Truck	Rating Factor (RF)	Rating (RT) (Tons)	Rating Type
HS20	1.700	61.2	Operating
HS20	1.019	36.7	Inventory
MO5	3.400	122.4	Operating
H20	2.924	58.5	Posting
3S2	2.924	107.1	Posting

\* All Units Expressed in English System

Table 39 - Rating Factor for the Central Span Slab (Negative Bending Moments)

Truck	Rating Factor (RF)	Rating (RT) (Tons)	Rating Type
HS20	2.888	104.0	Operating
HS20	1.730	62.3	Inventory
MO5	5.776	207.9	Operating
H20	4.967	99.3	Posting
3S2	4.967	182.0	Posting

\* All Units Expressed in English System

Table 40 - Rating Factor for the Central Span Slab (Shear)

Truck	Rating Factor (RF)	Rating (RT) (Tons)	Rating Type
HS20	1.396	50.3	Operating
HS20	0.836	30.1	Inventory
MO5	2.792	102.3	Operating
H20	2.401	48.0	Posting
3S2	2.401	88.0	Posting

\* All Units Expressed in English System

## ***D.2 Girders Rating***

The bending moment values due to the live loads for the exterior girders corresponding to the most critical sections are summarized below in Table 41 and

Table 42 for central and lateral spans respectively.

Table 43 and Table 44 summarize the corresponding rating factors.

Table 41 - Maximum Bending Moments due to the Live Loads at the Critical Positions for the Exterior Girders (Central Span)

Position [in]	Bending Moment at the Critical Positions [kip-ft]				Bending Moment at the Critical Positions with Impact Factor [kip-ft]			
	<b>HS20</b>	<b>MO5</b>	<b>H20</b>	<b>3S2</b>	<b>HS20</b>	<b>MO5</b>	<b>H20</b>	<b>3S2</b>
39	85.8	76.3	56.0	64.7	109.8	97.7	71.7	82.8
84	169.7	145.2	110.4	123.2	217.2	185.8	141.4	157.7
114	213.2	181.8	140.4	147.2	272.9	232.7	179.7	188.4
144	247.7	210.5	165.3	172.8	317.1	269.4	211.6	221.2
168	268.8	231.2	181.7	190.4	344.1	295.9	232.5	243.7
300	313.8	269.5	218.1	216.1	401.7	345.0	279.2	276.6

Table 42 - Maximum Bending Moments due to the Live Loads at the Critical Positions for the Exterior Girders (Lateral Spans)

Position [in]	Bending Moment at the Critical Positions [kip-ft]				Bending Moment at the Critical Positions with Impact Factor [kip-ft]			
	<b>HS20</b>	<b>MO5</b>	<b>H20</b>	<b>3S2</b>	<b>HS20</b>	<b>MO5</b>	<b>H20</b>	<b>3S2</b>
27	57.9	49.6	38.9	38.2	75.2	64.4	50.6	49.7
60	116.2	99.0	79.7	77.8	151.1	128.7	103.6	101.2
114	182.0	154.4	129.8	125.6	236.6	200.7	168.7	163.3
144	202.9	176.7	148.7	143.2	263.7	229.8	193.3	186.2
168	214.1	190.7	159.3	152.4	278.4	247.9	207.1	198.1
180	219.9	197.4	163.1	155.6	285.9	256.6	212.0	202.3
240	226.1	212.4	170.5	164.3	294.0	276.2	221.6	213.6

Table 43 – Rating Factors for the Exterior Girders of the Central Span (Bending Moments)

			Rating Factors $RF_i$ Computed at the Critical Positions				
Position [in]	Un-factored Moment due to Dead Load [k-ft]	Moment Capacity [k-ft]	HS20	HS20	MO5	H20	3S2
39	165.9	842.8	4.39	2.63	4.94	5.78	5.01
84	328.7	1280.9	3.02	1.81	3.53	3.99	3.58
114	420.1	1569.8	2.89	1.73	3.38	3.77	3.59
144	497.9	1820.0	2.85	1.70	3.35	3.67	3.51
168	550.3	1820.0	2.47	1.48	2.87	3.14	3.00
300	682.4	1820.0	1.79	1.07	2.08	2.21	2.23
Rating Factor: $RF = \min \{RF_i\}$			1.79	1.07	2.08	2.21	2.23
Rating (RT) [Tons]			64.32	38.53	76.22	44.22	81.75
Rating Type			Oper.	Invet.	Oper.	Posting	Posting

\* All Units Expressed in English System

Table 44 – Rating Factors for the Exterior Girders of the Lateral Spans (Bending Moments)

			Rating Factors $RF_i$ Computed at the Critical Positions				
Position [in]	Un-factored Moment due to Dead Load [k-ft]	Moment Capacity [k-ft]	HS20	HS20	MO5	H20	3S2
27	81.4	800.9	7.11	4.26	8.30	9.09	9.26
60	167.7	1035.9	4.16	2.49	4.89	5.22	5.35
114	277.7	1035.9	2.19	1.31	2.59	2.65	2.72
144	322.0	1035.9	1.80	1.08	2.07	2.03	2.03
168	348.9	1114.6	1.83	1.09	2.05	1.90	1.90
180	359.4	1200.9	1.97	1.18	2.20	1.99	1.99
240	383.4	1200.9	1.76	1.06	1.76	1.51	1.51
Rating Factor: $RF = \min \{RF_i\}$			1.76	1.06	1.76	1.51	1.51
Rating (RT) [Tons]			63.41	37.99	64.53	30.29	55.50
Rating Type			Oper.	Invet.	Oper.	Posting	Posting

\* All Units Expressed in English System

The shear force values due to the live loads for the exterior girders corresponding to the most critical sections are summarized below in Table 45 and Table 46 for central and

lateral

spans

respectively.

Table 47 and Table 48 summarize the corresponding rating factors.

Table 45 - Maximum Shear Forces at the Critical Positions due to Live Loads for the Exterior Girders (Central Span)

Position [in]	Shear Forces at the Critical Positions [kip]				Shear Forces at the Critical Positions with Impact Factor [kip]			
	<b>HS20</b>	<b>MO5</b>	<b>H20</b>	<b>3S2</b>	<b>HS20</b>	<b>MO5</b>	<b>H20</b>	<b>3S2</b>
0	29.3	25.2	18.4	22.5	37.5	32.3	23.5	28.8
18	27.8	24.4	18.0	21.3	35.6	31.2	23.0	27.3
36	27.1	23.6	17.3	20.5	34.7	30.2	22.1	26.2
84	24.2	20.7	14.8	15.7	31.0	26.6	18.9	20.1
102	23.2	19.7	13.8	15.0	29.7	25.2	17.6	19.2
200	17.3	14.3	13.0	11.7	22.1	18.4	16.6	15.0
300	10.6	10.0	8.6	8.0	13.6	12.8	11.0	10.2

Table 46- Maximum Shear Forces at the Critical Positions due to Live Loads for the Exterior Girders (Lateral Spans)

Position [in]	Shear Forces at the Critical Positions [kip]				Shear Forces at the Critical Positions with Impact Factor [kip]			
	<b>HS20</b>	<b>MO5</b>	<b>H20</b>	<b>3S2</b>	<b>HS20</b>	<b>MO5</b>	<b>H20</b>	<b>3S2</b>
0	27.8	23.1	18.3	18.1	36.2	30.1	23.7	23.5
18	26.4	22.2	17.7	17.3	34.3	28.9	23.0	22.6
36	25.0	21.3	17.0	16.6	32.5	27.7	22.0	21.5
60	23.2	19.5	15.9	15.5	30.2	25.4	20.7	20.2
132	17.8	15.1	12.9	12.4	23.2	19.6	16.8	16.1
240	9.8	9.7	8.3	7.7	12.8	12.6	10.8	10.0

Table 47 - Rating Factors for the Exterior Girders of the Central Span (Shear Forces)

			Rating Factors $RF_i$ Computed at the Critical Positions				
Position [in]	Un-factored Shear Forces due to Dead Load [kip]	Shear Capacity [kip]	HS20	HS20	MO5	H20	3S2
0	54.6	261.7	3.91	2.34	4.54	5.36	4.38
18	51.3	206.5	3.02	1.81	3.44	4.02	3.39
36	48.0	172.1	2.43	1.46	2.79	3.28	2.77
84	39.3	143.6	2.30	1.38	2.68	3.24	3.05
102	36.0	120.0	1.90	1.14	2.24	2.75	2.52
200	18.2	120.0	3.35	2.00	4.04	3.84	4.26
300	0.0	71.0	4.03	2.41	4.26	4.28	4.59
Rating Factor: $RF = \min \{RF_i\}$			1.90	1.14	2.24	2.75	2.52
Rating (RT) [Tons]			68.26	40.89	81.93	54.92	92.41
Rating Type			Oper.	Invet.	Oper.	Posting	Posting

\* All Units Expressed in English System

Table 48 - Rating Factors for the Exterior Girders of the Lateral Spans (Shear Forces)

			Rating Factors $RF_i$ Computed at the Critical Positions				
Position [in]	Un-factored Shear Forces due to Dead Load [kip]	Shear Capacity [kip]	HS20	HS20	MO5	H20	3S2
0	38.3	134.2	1.67	0.999	1.67	1.43	1.43
18	35.5	134.2	1.77	1.06	1.77	1.52	1.52
36	32.6	128.7	2.04	1.22	2.39	2.59	2.65
60	28.8	104.4	1.71	1.02	2.03	2.14	2.19
132	17.3	85.6	2.09	1.26	2.48	2.49	2.60
240	0.0	79.3	4.78	2.86	4.86	4.84	5.23
Rating Factor: $RF = \min \{RF_i\}$			1.67	0.999	1.67	1.43	1.43
Rating (RT) [Tons]			60.00	35.95	61.07	28.67	52.52
Rating Type			Oper.	Invet.	Oper.	Posting	Posting

\* All Units Expressed in English System

The bending moment values due to the live loads for the interior girders corresponding to the most critical sections are summarized below in Table 49 and

Table 50 for central and lateral spans respectively.

Table 51 and Table 52 summarize the corresponding rating factors.

Table 49 - Maximum Bending Moments due to the Live Loads at the Critical Positions for the Interior Girders (Central Span)

Position [in]	Bending Moment at the Critical Positions [kip-ft]				Bending Moment at the Critical Positions with Impact Factor [kip-ft]			
	HS20	MO5	H20	3S2	HS20	MO5	H20	3S2
48	179.3	155.8	115.3	134.5	229.5	199.4	147.6	172.2
96	319.5	246.5	208.9	225.5	409.0	315.5	267.4	288.6
114	362.0	308.8	238.4	250.0	463.4	395.2	305.1	320.0
144	420.6	357.4	280.7	293.4	538.4	457.5	359.3	375.6
168	456.4	392.5	308.5	323.2	584.2	502.5	394.9	413.7
204	493.8	432.3	339.9	349.3	632.1	553.3	435.1	447.1
300	532.5	457.6	370.3	367.0	681.6	585.7	474.0	469.8

Table 50 - Maximum Bending Moments due to the Live Loads at the Critical Positions for the Interior Girders (Lateral Spans)

Position [in]	Bending Moment at the Critical Positions [kip-ft]				Bending Moment at the Critical Positions with Impact Factor [kip-ft]			
	HS20	MO5	H20	3S2	HS20	MO5	H20	3S2
27	97.4	83.5	65.6	64.4	126.7	108.6	85.3	83.7
60	195.9	166.8	134.3	131.2	254.6	216.9	174.6	170.5
114	306.6	260.2	218.7	211.7	398.6	338.2	284.3	275.2
144	341.8	297.9	250.6	241.2	444.4	387.2	325.8	313.6
168	360.9	321.4	268.5	257.0	469.1	417.8	349.0	334.1
204	380.7	348.9	284.0	272.4	494.9	453.6	369.2	354.1
240	381.3	358.0	287.3	277.0	495.6	465.5	373.5	360.2



Table 51 – Rating Factors for the Interior Girders of the central span (Bending Moments)

			Rating Factors $RF_i$ Computed at the Critical Positions				
Position [in]	Un-factored Moment due to Dead Load [k-ft]	Moment Capacity [k-ft]	HS20	HS20	MO5	H20	3S2
48	172.2	849.5	2.097	1.256	2.413	2.804	2.404
96	314.5	1296.5	1.670	1.000	2.164	2.196	2.034
114	360.2	1591.9	1.865	1.118	2.187	2.436	2.323
144	426.8	1701.9	1.639	0.982	1.929	2.112	2.021
168	471.8	1913.6	1.712	1.026	1.991	2.178	2.079
204	525.1	2090.8	1.714	1.027	1.958	2.141	2.084
300	585.0	2220.0	1.647	0.987	1.917	2.037	2.055
Rating Factor: $RF = \min \{RF_i\}$			1.639	0.982	1.917	2.037	2.021
Rating (RT) [Tons]			59.00	35.35	70.23	40.74	74.03
Rating Type			Oper.	Invet.	Oper.	Posting	Posting

\* All Units Expressed in English System

Table 52 – Rating Factors for the Interior Girders of the Lateral Spans (Bending Moments)

			Rating Factors $RF_i$ Computed at the Critical Positions				
Position [in]	Un-factored Moment due to Dead Load [k-ft]	Moment Capacity [k-ft]	HS20	HS20	MO5	H20	3S2
27	70.1	813.6	4.388	2.629	5.117	5.606	5.712
60	144.4	1053.8	2.617	1.567	3.072	3.281	3.360
114	239.0	1302.2	1.913	1.146	2.255	2.307	2.384
144	277.2	1389.8	1.782	1.068	2.045	2.090	2.172
168	300.3	1519.7	1.852	1.109	2.079	2.141	2.236

204	322.6	1519.7	1.710	1.024	1.866	1.972	2.056
240	330.0	1519.7	1.693	1.014	1.803	1.932	2.003
Rating Factor: $RF = \min \{RF_i\}$			1.693	1.014	1.803	1.932	2.003
Rating (RT) [Tons]			60.94	36.51	66.05	38.64	73.40
Rating Type			Oper.	Invet.	Oper.	Posting	Posting

\* All Units Expressed in English System

The shear force values due to the live loads for the interior girders corresponding to the most critical sections are summarized below in Table 53 and Table 54 for central and lateral spans respectively. Table 55 and Table 56 summarize the corresponding rating factors.

Table 53 - Maximum Shear Forces at the Critical Positions due to Live Loads for the Interior Girders (Central Span)

Position [in]	Shear Forces at the Critical Positions [kip]				Shear Forces at the Critical Positions with Impact Factor [kip]			
	<b>HS20</b>	<b>MO5</b>	<b>H20</b>	<b>3S2</b>	<b>HS20</b>	<b>MO5</b>	<b>H20</b>	<b>3S2</b>
0	49.8	42.8	31.2	38.2	63.7	54.8	39.9	48.9
18	47.2	41.4	30.5	36.2	60.4	53.0	39.1	46.3
36	46.0	40.1	29.3	34.8	58.9	51.3	37.6	44.6
84	41.1	35.2	25.1	26.7	52.6	45.1	32.1	34.1
102	39.4	33.4	23.4	25.5	50.4	42.8	29.9	32.6
200	29.4	24.4	22.0	19.9	37.6	31.2	28.2	25.4
300	18.0	17.0	14.6	13.6	23.0	21.8	18.6	17.4

Table 54- Maximum Shear Forces at the Critical Positions due to Live Loads for the Interior Girders (Lateral Spans)

Position [in]	Shear Forces at the Critical Positions [kip]				Shear Forces at the Critical Positions with Impact Factor [kip]			
	<b>HS20</b>	<b>MO5</b>	<b>H20</b>	<b>3S2</b>	<b>HS20</b>	<b>MO5</b>	<b>H20</b>	<b>3S2</b>
0	46.8	39.0	30.8	30.5	60.9	50.7	40.0	39.6
18	44.5	37.5	29.9	29.3	57.8	48.7	38.8	38.1
36	42.3	36.0	28.6	28.0	55.0	46.7	37.2	36.4
60	39.2	32.9	26.9	26.2	51.0	42.8	34.9	34.1
132	30.0	25.4	21.7	21.0	39.0	33.0	28.3	27.3
240	16.6	16.3	14.1	13.1	21.6	21.2	18.3	17.0

Table 55 - Rating Factors for the Interior Girders of the Central Span (Shear Forces)

Position [in]	Un-factored Shear Forces due to Dead Load [kip]	Shear Capacity [kip]	Rating Factors $RF_i$ Computed at the Critical Positions				
			<b>HS20</b>	<b>HS20</b>	<b>MO5</b>	<b>H20</b>	<b>3S2</b>
0	46.8	261.7	2.701	1.454	2.819	2.643	2.643
18	44.0	206.5	2.169	1.139	2.165	1.990	1.990
36	41.2	188.9	2.007	1.059	2.031	1.829	1.829
84	35.6	174.1	2.086	1.120	2.181	2.633	2.478
102	33.7	160.4	1.987	1.066	2.097	2.576	2.366
200	15.6	117.3	1.940	1.189	2.395	2.276	2.524
300	0.0	117.3	3.456	2.346	4.143	4.163	4.463
Rating Factor: $RF = \min \{RF_i\}$			1.940	1.059	2.031	1.829	1.829
Rating (RT) [Tons]			69.83	38.12	74.40	36.59	67.03
Rating Type			Oper.	Invet.	Oper.	Posting	Posting

\* All Units Expressed in English System

Table 56 - Rating Factors for the Interior Girders of the Lateral Spans (Shear Forces)

Position [in]	Un-factored Shear Forces due to Dead Load [kip]	Shear Capacity [kip]	Rating Factors $RF_i$ Computed at the Critical Positions				
			<b>HS20</b>	<b>HS20</b>	<b>MO5</b>	<b>H20</b>	<b>3S2</b>
0	33.0	175.8	1.771	1.006	1.949	1.676	1.676
18	30.5	171.6	1.834	1.052	1.994	1.715	1.715
36	28.0	164.0	1.844	1.069	2.100	2.272	2.320
60	24.7	134.3	1.576	0.924	1.837	1.936	1.984
132	14.8	97.4	1.471	0.924	1.821	1.830	1.897
240	0.0	90.4	2.775	1.932	3.286	3.274	3.509

Rating Factor: $RF = \min \{RF_i\}$	1.471	0.924	1.821	1.676	1.676
Rating (RT) [Tons]	69.83	38.12	74.40	36.59	67.03
Rating Type	Oper.	Invet.	Oper.	Posting	Posting

\* All Units Expressed in English System

### D.3 Bents

Table 57 summarizes the vertical forces due to the live loads transferred to the bents corresponding to the possible load combinations of the girders.

Table 58 summarizes ultimate moments and shear forces calculated at four cross-sections where maximum values are reached.

Table 57 – Load to the Bent due to the Live Load (Including Impact Factors)

Loading Conditions	Force to the Bent [kip]	HS-20	H-20	MO-5	3S2
<i>Ia)</i>	P <sub>1L</sub>	40.6	33.7	34.4	30.2
	P <sub>L</sub>	43.9	36.9	37.1	32.6
	P <sub>2L</sub>	-4.0	-3.0	-2.2	-2.5
<i>Ib)=IIa)</i>	P <sub>1L</sub>	23.5	19.9	19.9	17.5
	P <sub>L</sub>	59.7	49.6	50.5	44.3
	P <sub>2L</sub>	-2.8	-2.1	-1.6	-1.8
<i>IIb)</i>	P <sub>1L</sub>	5.8	5.0	4.9	4.4
	P <sub>L</sub>	68.6	57.1	57.9	50.9
	P <sub>2L</sub>	5.8	5.0	4.9	4.4

Table 58 – Bending Moments and Shear Forces at the Critical Cross Sections

Sections	Bending Moment at the Critical Positions [kip-ft/ft]				Shear Forces at the Critical Positions [kip-ft/ft]			
	HS20	H20	3S2	MO5	HS20	H20	3S2	MO5
AA	87.0	72.2	64.5	83.3	40.6	33.7	30.1	34.4
BB	126.5	126.5	126.5	126.5	44.9	44.9	44.9	44.9
CC	243.9	243.9	243.9	243.9	44.9	44.9	44.9	44.9
DD	109.9	109.9	109.9	109.9	6.9	6.9	6.9	6.9

Table 59 - Rating Factors for the Bents (Bending Moments)

			Rating Factors $RF_i$ Computed at the Critical Positions				
Sections	Un-factored Bending Moment due to Dead Load [k-ft]	Moment Capacity [k-ft]	HS20	HS20	MO5	H20	3S2
AA	88.2	497.5	3.386	2.028	3.536	3.509	3.926
BB	198.1	497.5	1.460	0.874	1.460	1.255	1.255
CC	368.7	1020.4	1.707	1.022	1.707	1.468	1.468
DD	109.9	400.0	1.799	1.078	1.799	1.548	1.548
Rating Factor: $RF = \min \{RF_i\}$			1.460	0.874	1.460	1.255	1.255
Rating (RT) [Tons]			52.54	31.48	53.48	25.10	45.99
Rating Type			Operating	Inventory	Operating	Posting	Posting

\* All Units Expressed in English System

Table 60 - Rating Factors for the Bents (Shear Forces)

			Rating Factors $RF_i$ Computed at the Critical Positions				
Sections	Un-factored Shear Forces due to Dead Load [kip]	Shear Capacity [kip]	HS20	HS20	MO5	H20	3S2
AA	41.2	176.1	2.320	1.390	2.744	2.404	2.690
BB	68.7	176.1	1.487	0.891	1.487	1.279	1.279
CC	68.7	176.1	1.487	0.891	1.487	1.279	1.279
DD	10.0	98.7	9.586	5.743	9.586	8.244	8.244
Rating Factor: $RF = \min \{RF_i\}$			1.487	0.891	1.487	1.279	1.279
Rating (RT) [Tons]			53.54	32.07	54.49	25.58	46.86
Rating Type			Operating	Inventory	Operating	Posting	Posting

\* All Units Expressed in English System

#### **D.4 Piers**

Axial loads due to live loads and corresponding rating factors are summarized in Table 61 and Table 62.

Table 61 - Axial Loads due to Live Loads

Truck	Maximum Axial Load [kip]	Maximum Axial Load with Impact Factor [kip]
HS20	105.6	137.3
MO5	105.6	137.3
H20	105.6	137.3
3S2	105.6	137.3

Table 62 - Rating Factor for the Piers (Axial Loads)

Truck	Rating Factor (RF)	Rating (RT) (Tons)	Rating Type
HS20	5.756	207.2	Operating
HS20	3.449	124.1	Inventory
MO5	5.756	207.2	Operating
H20	4.951	99.0	Posting
3S2	4.951	181.4	Posting

\* All Units Expressed in English System

#### **D.5 Summary and Conclusions**

The rating of the bridge is determined by the least rated element.

**Table 63** summarizes the rating of each element of the bridge. The most deficient element is the deck of the lateral spans for shear forces.

Since the factors RF with which posting is determined are greater than 1 the bridge does not need to be load posted. In addition, from

**Table 63** the maximum operating and inventory load can be found as 50.3 T and 30.1 T respectively.



Table 63 – Summary of the rating of all the elements

Elements	Rating Factors $RF_E$ for the Elements				
	HS20	HS20	MO5	H20	3S2
Slab Central Span (Positive Bending Moments)	1.731	1.037	3.462	1.708	1.708
Slab Central Span (Negative Bending Moments)	3.478	2.084	6.956	4.110	4.110
Slab Central Span (Shear Forces)	1.465	0.878	1.824	1.569	1.569
Slab Lateral Spans (Positive Bending Moments)	1.700	1.019	3.400	2.924	2.924
Slab Lateral Spans (Negative Bending Moments)	2.888	1.730	5.776	4.967	4.967
Slab Lateral Spans (Shear Forces)	1.396	0.836	2.792	2.401	2.401
Exterior Girders Central Span (Bending Moments)	1.787	1.070	2.080	2.211	2.231
Exterior Girders Central Span (Shear Forces)	1.896	1.136	2.236	2.746	2.522
Interior Girders Central Span (Bending Moments)	1.639	0.982	1.917	2.037	2.021
Interior Girders Central Span (Shear Forces)	1.940	1.059	2.031	1.829	1.829
Exterior Girders Lateral Spans (Bending Moments)	1.761	1.055	1.761	1.515	1.515
Exterior Girders Lateral Spans (Shear Forces)	1.667	0.999	1.667	1.433	1.433
Interior Girders Lateral Spans (Bending Moments)	1.693	1.014	1.803	1.932	2.003
Interior Girders Lateral Spans (Shear Forces)	1.471	0.924	1.821	1.676	1.676
Bents (Bending Moments)	1.460	0.874	1.460	1.255	1.255
Bents (Shear Forces)	1.487	0.891	1.487	1.279	1.279
Piers	5.756	3.449	5.756	4.951	4.951
Rating Factor: $RF = \min \{RF_E\}$	1.396	0.836	1.460	1.255	1.255
Rating (RT) [Tons]	50.25	30.11	53.48	25.10	45.99
Rating Type	Operating	Inventory	Operating	Post-ing	Post-ing

## REFERENCES

- <sup>1</sup> AASHTO, 2002: “Standard Specifications for Highway Bridges”, 17<sup>th</sup> Edition, Published by the American Association of State Highway and Transportation Officials, Washington D.C.
- <sup>2</sup> ACI 440.2R-02, 2002: “Guide for the Design and Construction of Externally Bonded FRP Systems for Strengthening Concrete Structures,” Published by the American Concrete Institute, Farmington Hills, MI.
- <sup>3</sup> AASHTO, 1998: “LRFD Bridge Design Specifications”, Second Edition, Published by the American Association of State Highway and Transportation Officials, Washington D.C.
- <sup>4</sup> ACI 318-99, 1999: “Building Code Requirements for Structural Concrete and Commentary (318R-99),” Published by the American Concrete Institute, Farmington Hills, MI.
- <sup>5</sup> AASHTO, 1996 “LRFD Design Code for Highway Bridges”, Published by the American Association of State Highway and Transportation Officials, Washington, D.C.
- <sup>6</sup> AASHTO, 1994 “Manual for Condition Evaluation of Bridges”, Published by the American Association of State Highway and Transportation Officials, Washington, D.C.

## APPENDIX A – Load Transfer and Slab Analysis

The statically indeterminate structure shown in Figure 7-I can be reduced to two simpler structures as represented in Figure 36. The vertical reaction  $R_2$  represents the unknowns of the problem to be determined by imposing the compatibility of the displacements as expressed in Eq. (17).

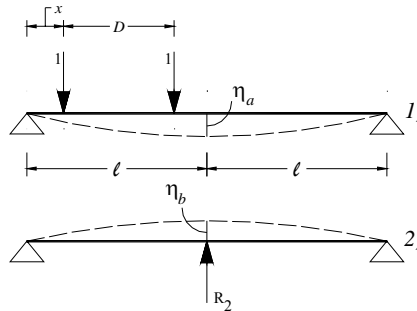


Figure 36 – Structures Equivalent to Figure 7-I

$$\begin{aligned} \eta_a &= \eta_b \\ \eta_b &= \frac{R_2 \ell^3}{6EI} \end{aligned} \tag{17}$$

By using superposition, Beam 1 in Figure 36 is equivalent to the two beams shown in Figure 37. The compatibility equation can be rearranged as follows:

$$\begin{aligned} \eta_a &= \eta_{a-1} + \eta_{a-2} \\ \eta_{a-1} &= \frac{x}{12EI} (3\ell^2 - x^2) \\ \eta_{a-2} &= \frac{x+D}{12EI} (3\ell^2 - (x+D)^2) \end{aligned} \tag{18}$$

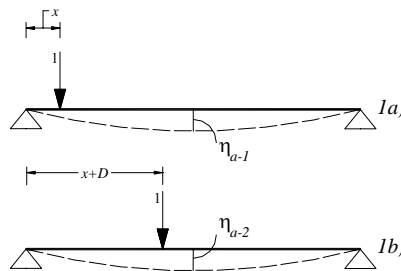


Figure 37 – Structures Equivalent to Beam 1 in Figure 36

The unknown  $R_2$  can be determined as follows:

$$R_2 = \frac{6}{\ell^3}(\eta_{a-1} + \eta_{a-2}) \quad (19)$$

Bending moment and shear force can be found from the following Eq. (20) (see Figure 38). It should be noted that the vertical reactions from Eq. (19) needs to be multiplied by  $P/2$  ( $P=$ axle load) because the previous analysis was conducted using unit forces.

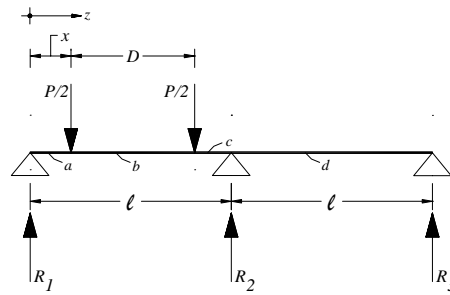


Figure 38 – Definitions for M and V

$$\begin{aligned} M_a &= R_1 z \\ M_b &= M_a - 0.5P(z - x) \\ M_c &= M_b - 0.5P[z - (x + D)] \\ M_d &= M_c + R_2(z - \ell) \\ V_a &= R_1 \\ V_b &= V_a - 0.5P \\ V_c &= V_b - 0.5P \\ V_d &= V_c + R_2 \end{aligned} \quad (20)$$

The case shown in Figure 7-II is similar to the one already presented if:

$$\begin{aligned} \eta_a &= \eta_{a-1} + \eta_{a-2} \\ \eta_{a-1} &= \frac{x}{12EI} (3\ell^2 - x^2) \\ \eta_{a-2} &= \frac{2\ell - (x + D)}{12EI} (3\ell^2 - [2\ell - (x + D)]^2) \end{aligned} \quad (21)$$

Bending moment and shear force for this second case can be written as follows:

$$\begin{aligned}M_a &= R_1 z \\M_b &= M_a - 0.5P(z - x) \\M_c &= M_b + R_2(z - \ell) \\M_d &= M_c - 0.5P[z - (x + D)] \\V_a &= R_1 \\V_b &= V_a - 0.5P \\V_c &= V_b - R_2 \\V_d &= V_c - 0.5P\end{aligned}\tag{22}$$

## APPENDIX B – Load Lane Analysis

### a) Distributed Load

As stated in AASHTO, the load lane load consists of  $q=0.64 \text{ kip/ft}$ , uniformly distributed in the longitudinal direction. Transversely, it is uniformly distributed over a  $10.0 \text{ ft}$  width. The share that each girder carries can be found by analyzing the structure shown in Figure 39.

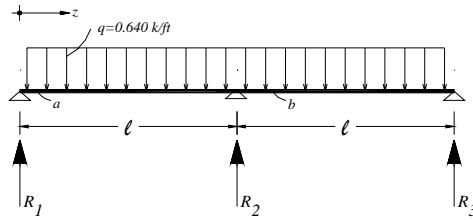


Figure 39 – Determination of  $k_\ell$

The beam represented in Figure 39 can be analyzed by removing the central support and imposing the compatibility equation. The vertical reactions can be written as follows:

$$\begin{aligned} R_1 = R_3 &= \frac{3}{8} q\ell \\ R_2 &= \frac{5}{4} q\ell \end{aligned} \tag{23}$$

The values of  $k_\ell$  are  $1.25$  and  $0.375$  for interior and exterior girders, respectively. Bending moment and shear force can be expressed as follows:

$$\begin{aligned} M_a &= \frac{3}{8} q\ell z - q \frac{z^2}{2} \\ M_b &= \frac{3}{8} q\ell z - q \frac{z^2}{2} + \frac{5}{4} q\ell (z - \ell) \\ V_a &= \frac{3}{8} q\ell - qz \\ V_b &= \frac{3}{8} q\ell - qz + \frac{5}{4} q\ell \end{aligned} \tag{24}$$

When only half of the deck is loaded with the uniform distribution  $q$ , the  $k_\ell$  coefficients are  $0.438$  and  $0.625$  for the external loaded reaction and internal reaction, respectively.

## APPENDIX C – Girder Analysis for an HS20-44 Truck

As previously recognized, girder analysis is carried out by taking into consideration only three of the five cases corresponding to five different positions of the design truck on the single span, as shown in Figure 13. The first case of Figure 13 is enlarged in Figure 40.

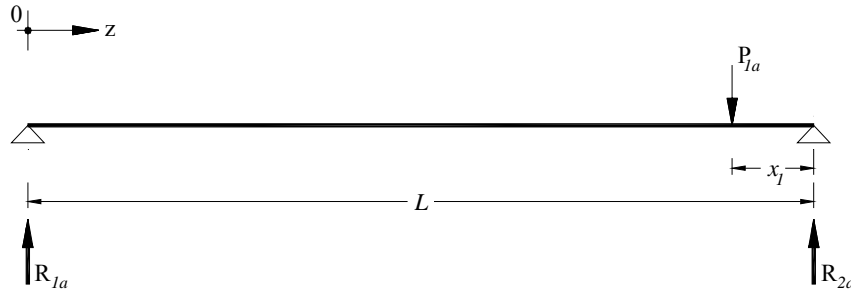


Figure 40 – One Wheel Load on the Girder

Vertical reactions  $R_1$  and  $R_2$  are defined as follows:

$$\begin{aligned} R_{1a} &= P_{1a} - R_{2a} \\ R_{2a} &= \frac{P_{1a}(L - x_1)}{L} \end{aligned} \quad (25)$$

Shear and moment diagrams can be expressed as a function of  $z$  as follows:

$$\begin{aligned} V_a(z) &= \begin{cases} R_{1a} & \text{if } z \leq L - x_1 \\ R_{1a} - P_{1a} & \text{otherwise} \end{cases} \\ M_a(z) &= \begin{cases} R_{1a}z & \text{if } z \leq L - x_1 \\ R_{1a}z - P_{1a}[z - (L - x_1)] & \text{otherwise} \end{cases} \end{aligned} \quad (26)$$

The second case (Figure 13b) is shown in Figure 41. Vertical reactions are:

$$\begin{aligned} R_{1b} &= P_{1a} + P_{1b} - R_{2b} \\ R_{2b} &= R_{2a} + \frac{P_{1b}(L - x_1 + L_{1a})}{L} \end{aligned} \quad (27)$$

Shear and moments can be written as:



$$V_b(z) = \begin{cases} R_{1b} & \text{if } z \leq L - x_1 \\ R_{1b} - P_{1a} & \text{if } L - x_1 \leq z \leq L - x_1 + L_{1a} \\ R_{1b} - P_{1a} - P_{1b} & \text{otherwise} \end{cases} \quad (28)$$

$$M_b(z) = \begin{cases} R_{1b}z & \text{if } z \leq L - x_1 \\ R_{1b}z - P_{1a}[z - (L - x_1)] & \text{if } L - x_1 \leq z \leq L - x_1 + L_{1a} \\ R_{1b}z - P_{1a}[z - (L - x_1)] + P_{1b}[z - (L - x_1 + L_{1a})] & \text{otherwise} \end{cases}$$

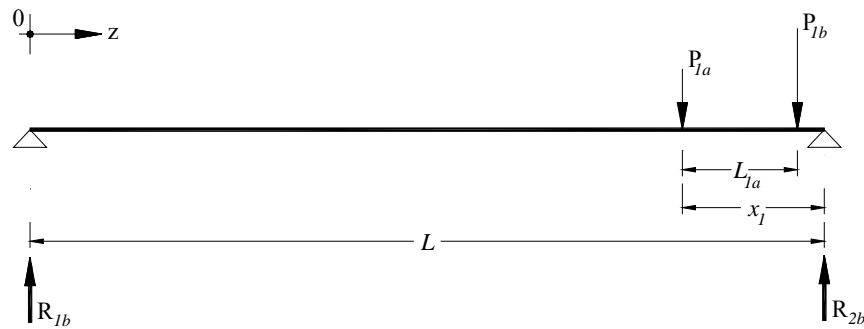


Figure 41 – Two Wheel Loads on the Girder

When three wheel loads are present on the girder (see Figure 42), vertical reactions are expressed as follows:

$$R_{1c} = P_{1a} + P_{1b} + P_{1c} - R_{2c}$$

$$R_{2c} = R_{2b} + \frac{P_{1c}(L - x_1 + L_{1a} + L_{1b})}{L} \quad (29)$$

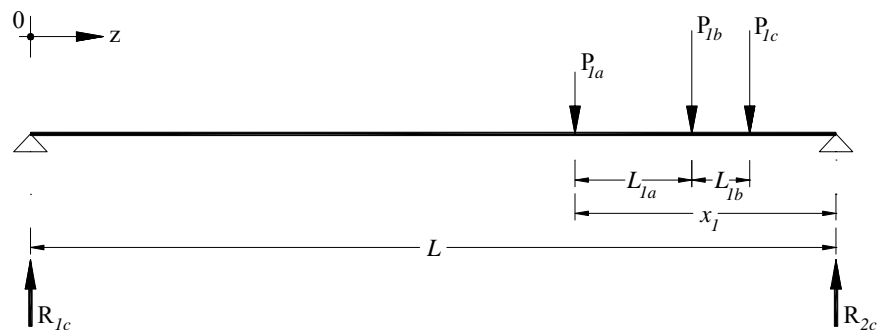


Figure 42 – Three Wheel Loads on the Girder

Shear and moments can be written as follows:

$$V_c(z) = \begin{cases} R_{1c} & \text{if } z \leq L - x_1 \\ R_{1c} - P_{1a} & \text{if } L - x_1 \leq z \leq L - x_1 + L_{1a} \\ R_{1c} - P_{1a} - P_{1b} & \text{if } L - x_1 + L_{1a} < z < L - x_1 + L_{1a} + L_{1b} \\ R_{1c} - P_{1a} - P_{1b} - P_{1c} & \text{otherwise} \end{cases} \quad (30)$$

$$M_c(z) = \begin{cases} R_{1c}z & \text{if } z \leq L - x_1 \\ R_{1c}z - P_{1a}[z - (L - x_1)] & \text{if } L - x_1 \leq z \leq L - x_1 + L_{1a} \\ R_{1c}z - P_{1a}[z - (L - x_1)] - P_{1b}[z - (L - x_1 + L_{1a})] & \text{if } L - x_1 + L_{1a} < z < L - x_1 + L_{1a} + L_{1b} \\ R_{1c}z - P_{1a}[z - (L - x_1)] - P_{1b}[z - (L - x_1 + L_{1a})] - P_{1c}[z - (L - x_1 + L_{1a} + L_{1b})] & \text{otherwise} \end{cases}$$

## APPENDIX D – Bent Analysis

The bent represented in Figure 19 can be considered equivalent to the the structures shown in Figure 43.

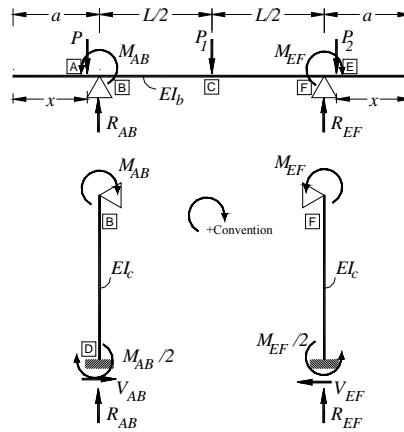


Figure 43 – Bent Equivalent Structures (Live Load)

The compatibility equation can be written as follows:

$$\alpha_{B\_beam} = \alpha_{B\_column} \quad (31)$$

$$\alpha_{F\_beam} = \alpha_{F\_column}$$

where:

$$\begin{aligned} \alpha_{beam} &= \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 \\ \alpha_1 &= \frac{-P(a-x)L}{3EI_b} \\ \alpha_2 &= \frac{-M_{AB}L}{3EI_b} \\ \alpha_3 &= \frac{P_1L^2}{16EI_b} \\ \alpha_4 &= \frac{-M_{EF}L}{6EI_b} \\ \alpha_5 &= \frac{-P_2(a-x)L}{6EI_b} \end{aligned} \quad (32)$$

and:

$$\alpha_{B\_column} = \frac{M_{AB}h}{4EI_c} \quad (33)$$

Similar equations can be written to meet the compatibility at F (Figure 43). By resolving the previous equation, the unknown  $M_{AB}$  can be determined.